Functional Thinking

Problem Solving in the Digital Age

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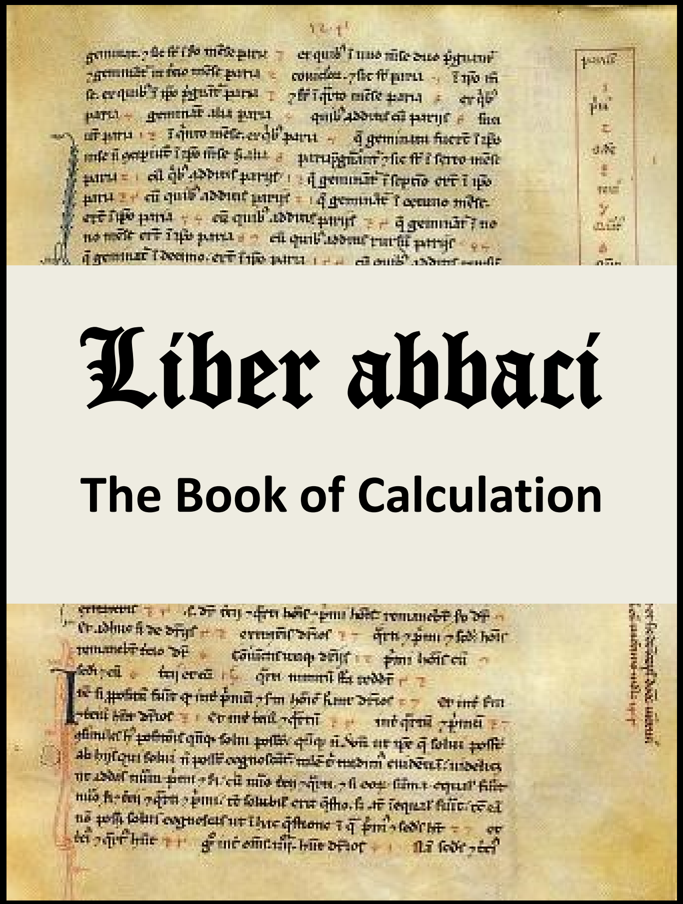
The problem solving we are taught today was designed in the year 1202

Leonardo of Pisa (c.1170-1250)

When Leonardo of Pisa…

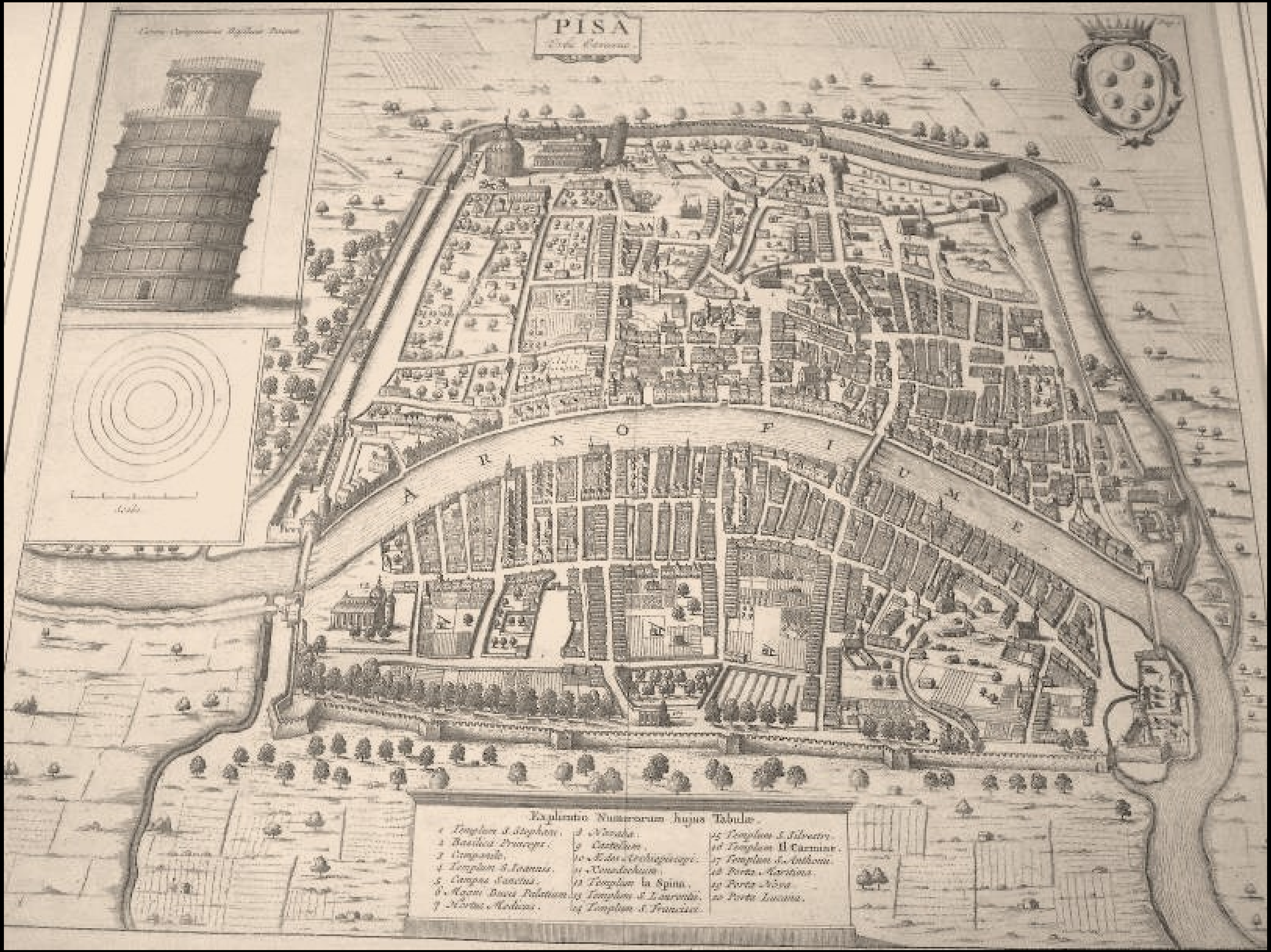
Published this book



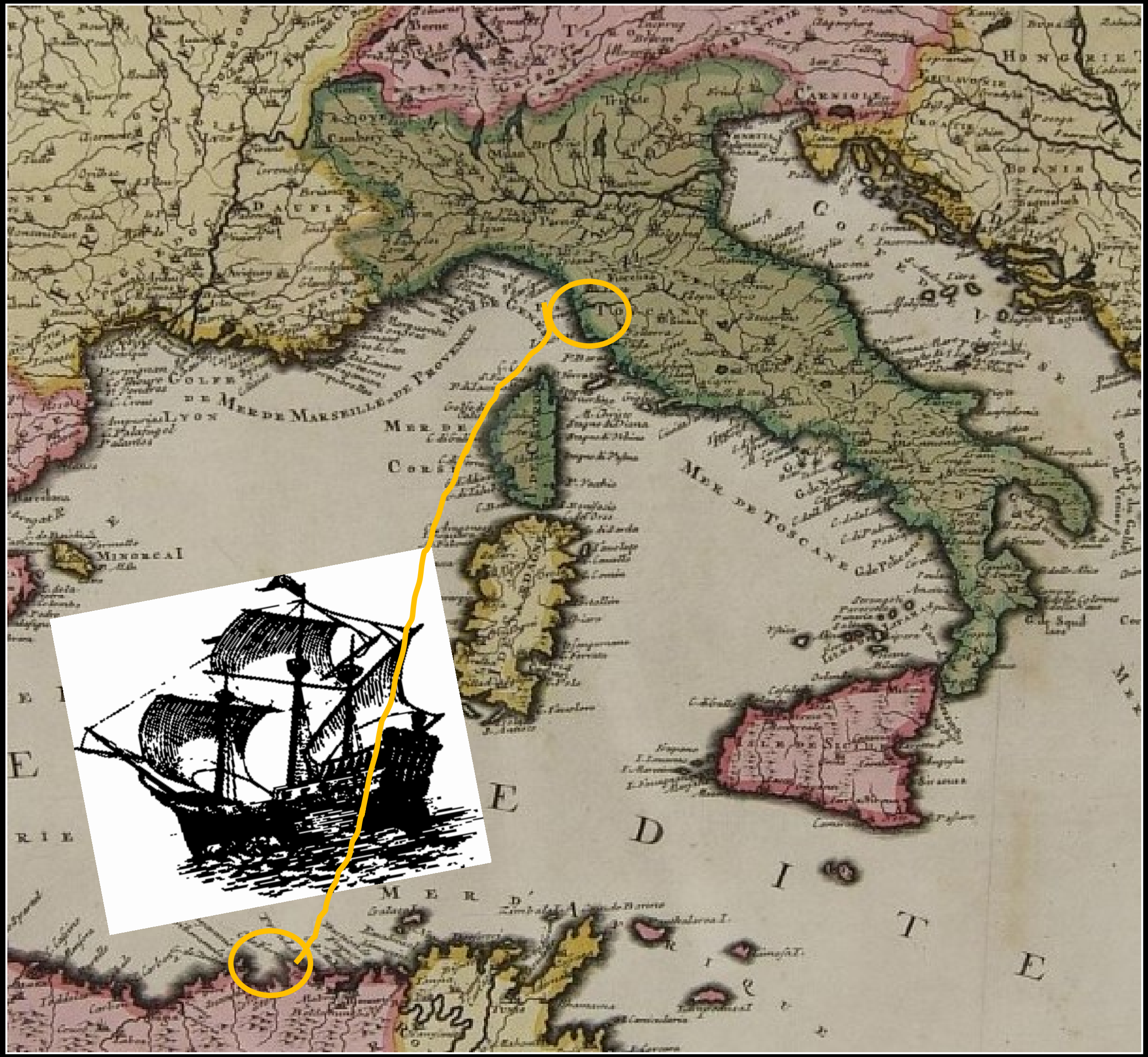
Born in Pisa

At the same time as the Leaning Tower

When Pisa was a great trading city



Leonardo joined his father, a “public official” and trader in Algeria



Where he was tutored in
Arabic arithmetic and algebra

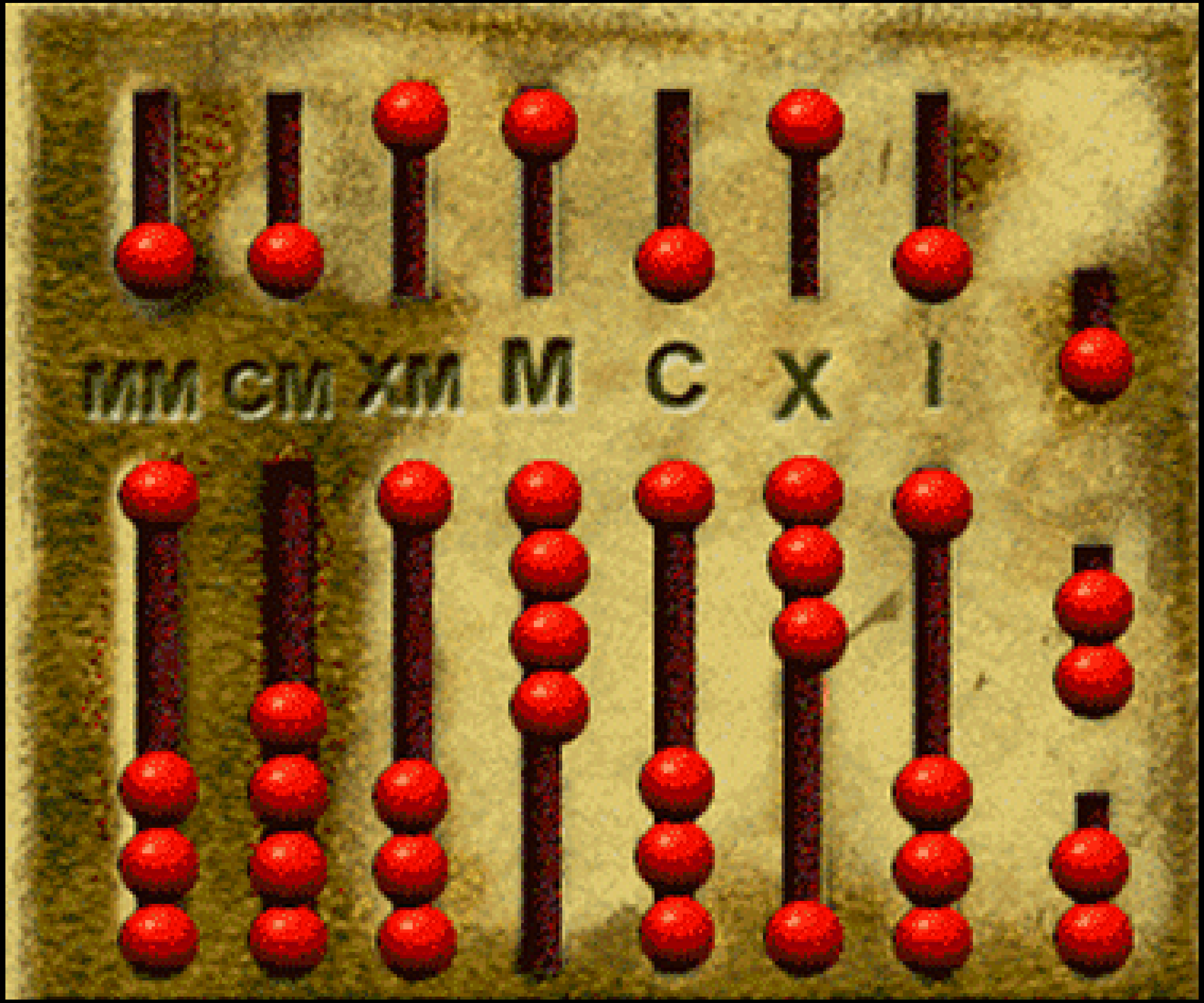
The Compendious Book on Calculation by Completion and Balancing
al Khwarizmi

Both academic subjects…

Scholars at an Abbasid library, Baghdad (1237)

…not used by
medieval merchants

Who computed
in Roman math
on an abacus

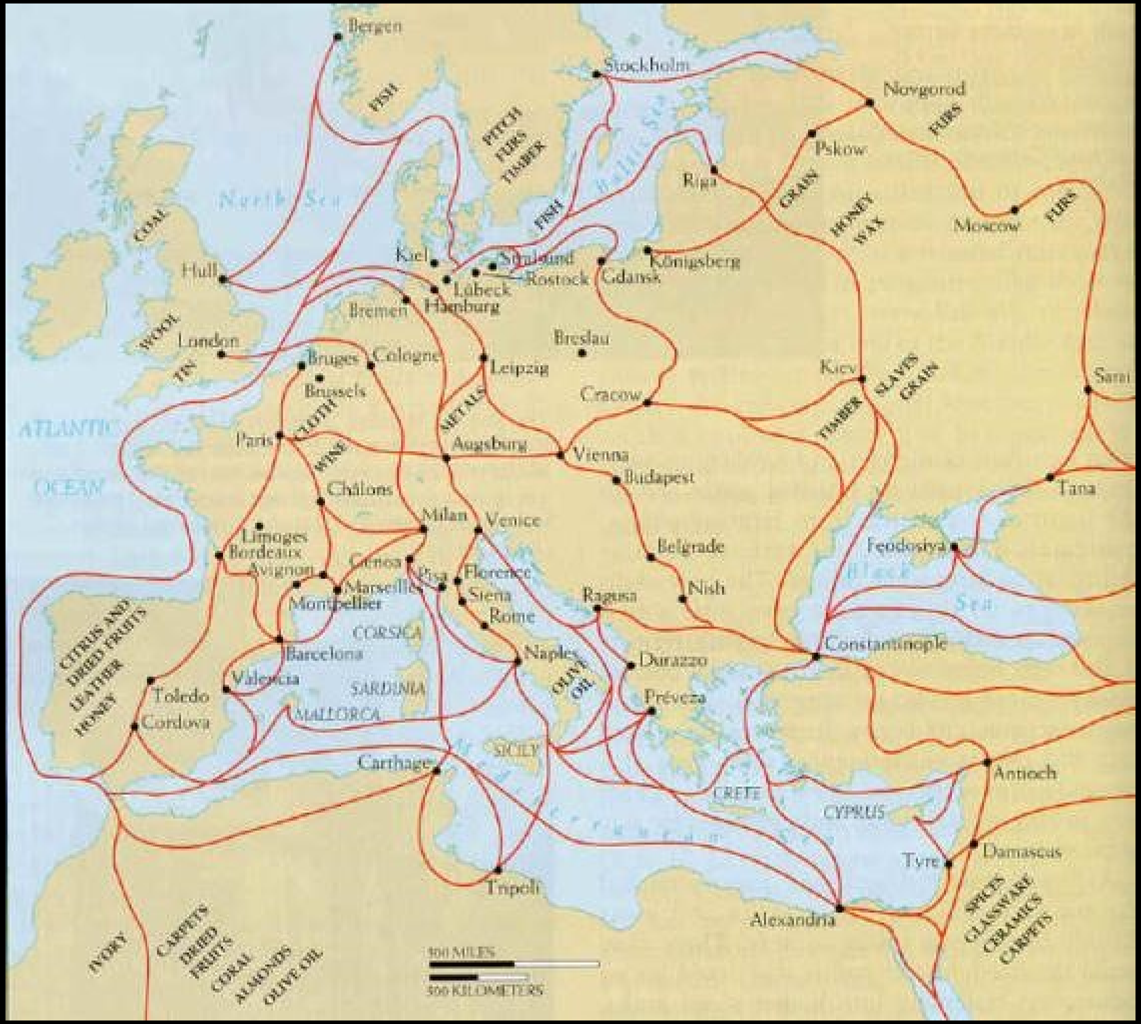


Good enough for the Roman Empire

"None of the cities should be allowed to have its own separate coinage or a system of weights and measures; they should all be required to use ours."

Dio Cassius 235AD

But not for trade between Medieval city-states, each with its own…



…weights,
measures,
and money

Requiring multiplication, division and ratio and proportion logic

CLXVII	**I**
CLXVIICLXVII	II
CLXVIICLXVIICLXVIICLXVII	IV
CLXVIICLXVIICLXVIICLXVII CLXVIICLXVIICLXVIICLXVII	VIII
CLXVII CLXVIICLXVIICLXVIICLXVII CLXVIICLXVIICLXVIICLXVII	I+VIII=IX

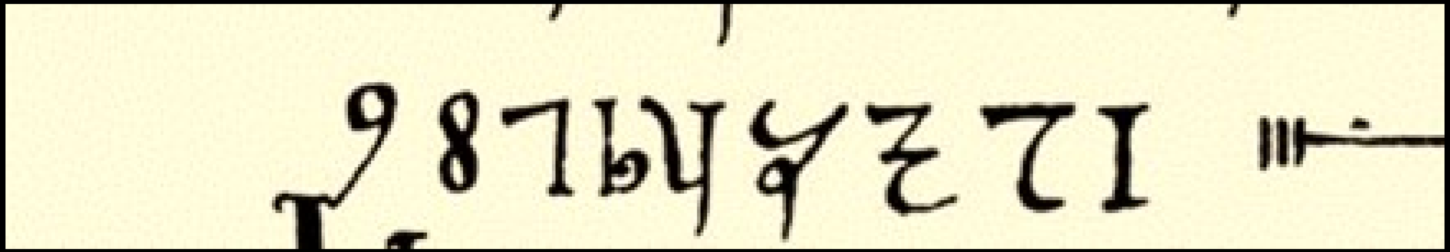
Roman multiplication by Doubling (167 x 9 =)

In 1200 Leonardo, a merchant himself, returned to Pisa to write the arithmetic necessary to merchants

The Liber Abaci or Book of Calculation is by Leonardo Pisano Bigollo or Fibonacci, considered by many as one of the most talented western mathematicians of the Middle Ages. Featured in a rare 15th century manuscript estimated at between $120,000-180,000 (£75,000 – 110,000). Photo: by Bonhams.

Liber abbaci

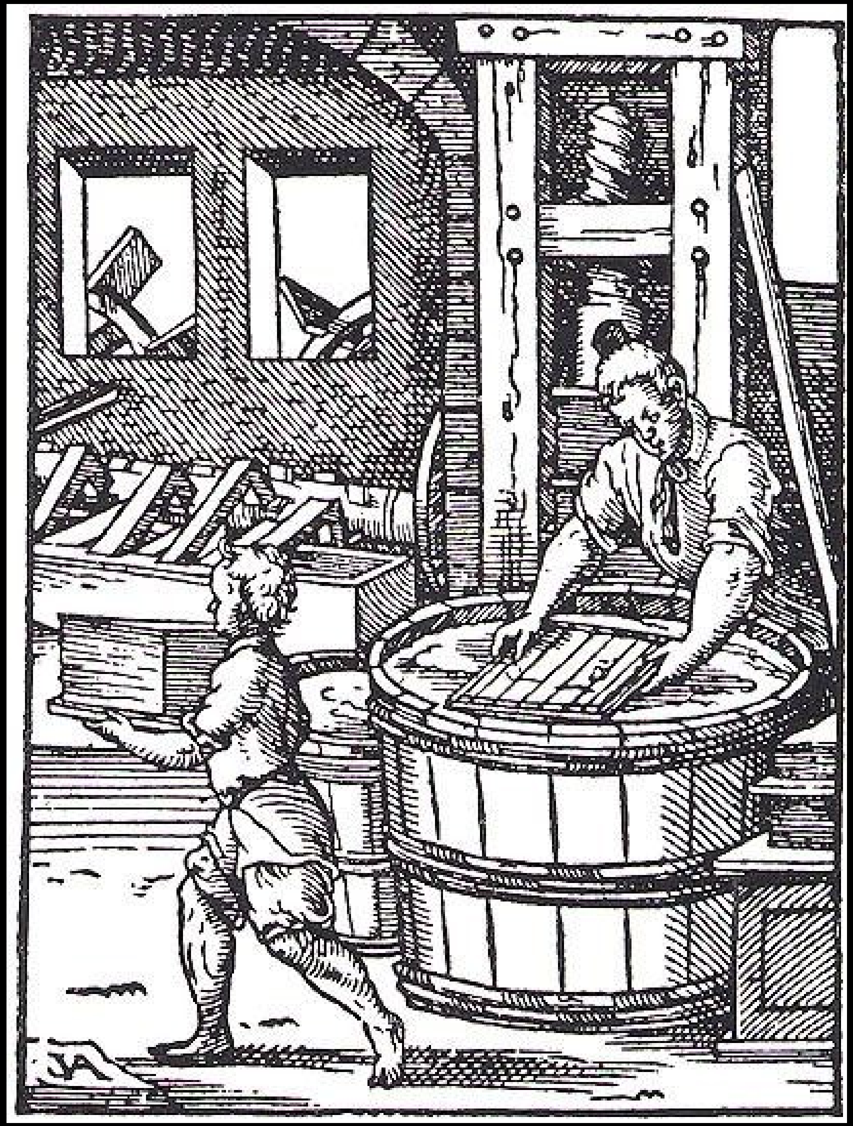
Based on Indian
numerals, place value
and…



al Khwarizmi's
“algorithmic”
procedures in
arithmetic and
algebra

al Khwarizmi (c780-c850)

Using not an abacus but
a new technology…
paper

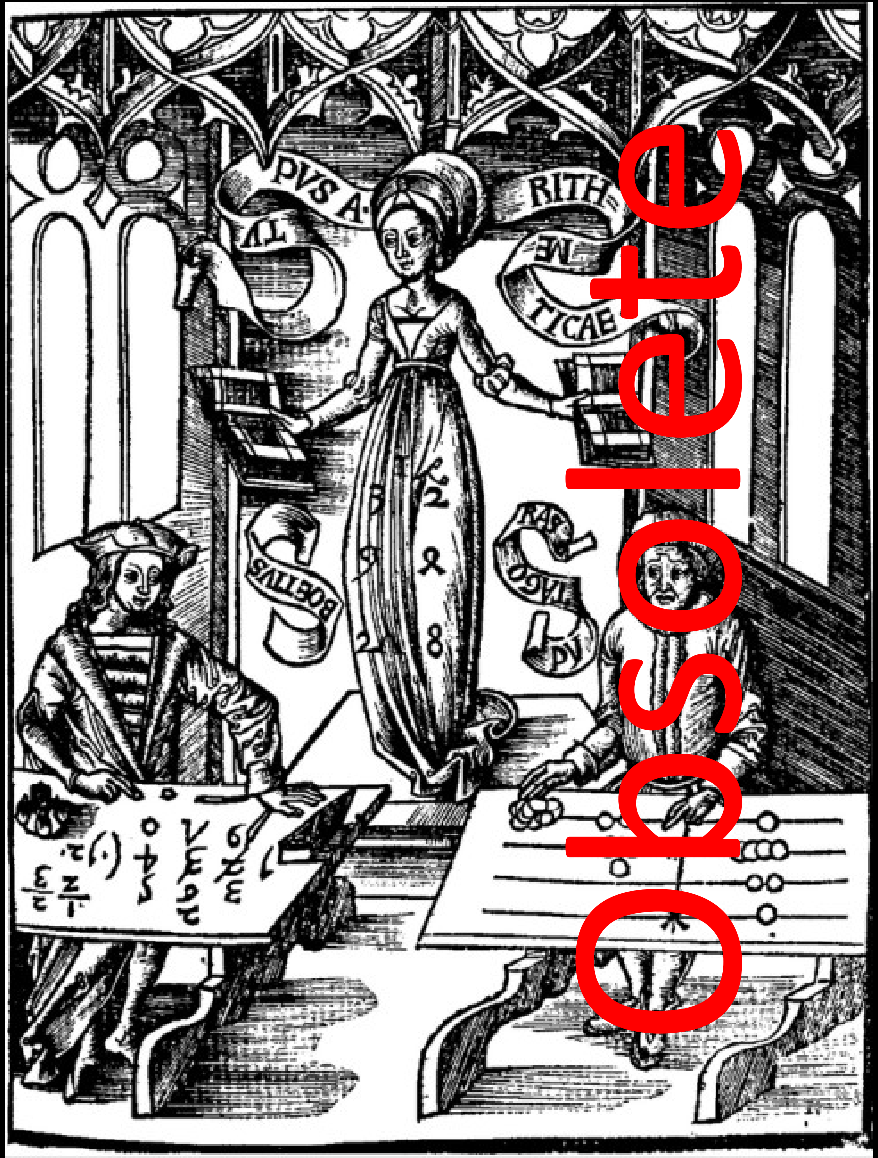


Paper introduced to Europe c. 1100

Leonardo's (algorist) math gradually become symbolic and made

Algorist vs. Abacist (woodcut 1504)

Leonardo's (algorist) math
gradually become symbolic
and made
Roman (abacist) math...



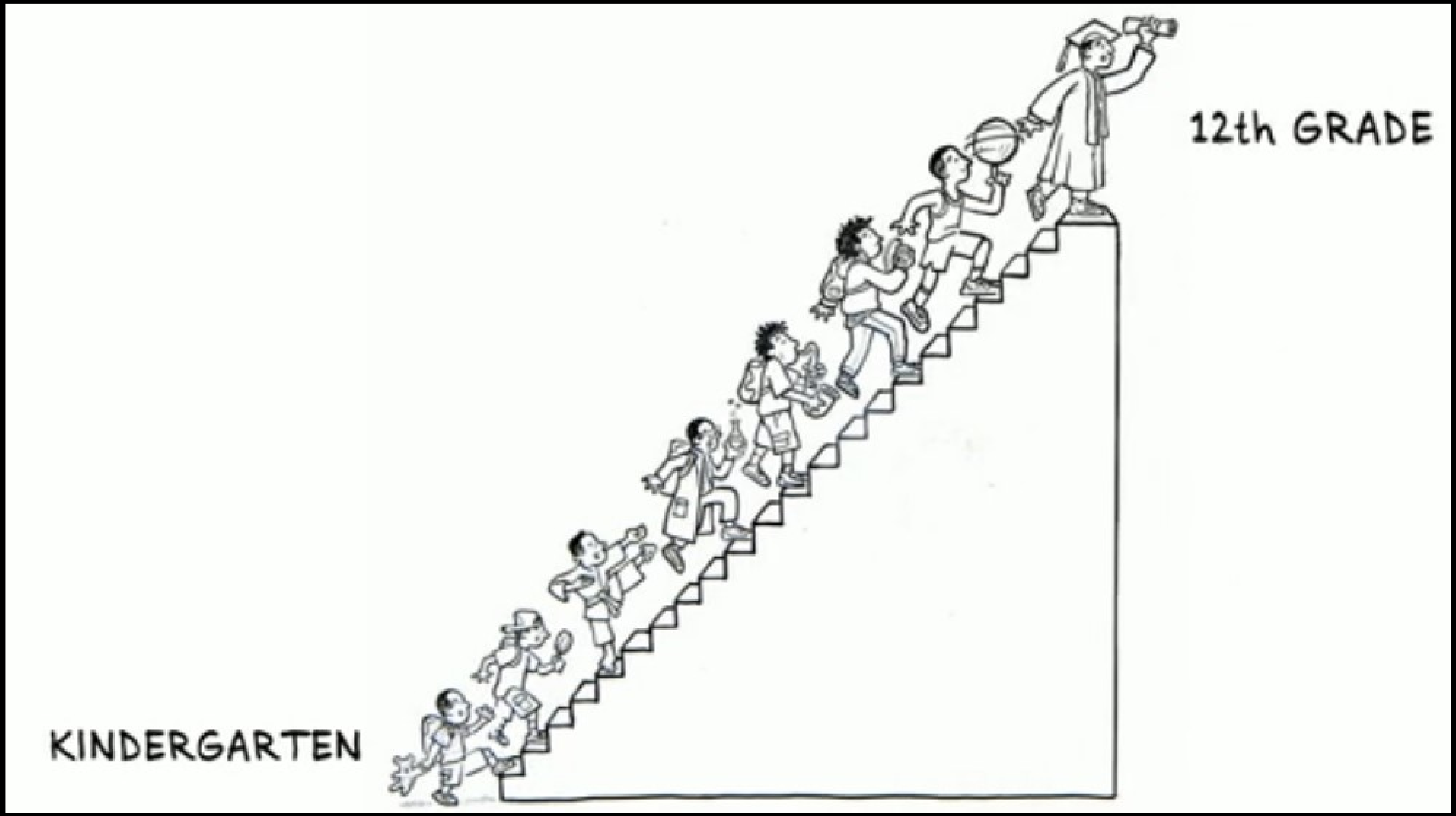
Algorist vs. Abacist (woodcut 1504)

By the 17th century Leonardo's table of contents…

1. *On the recognition of the nine Indian figures and how all numbers are written with them. (place value)*
2. *On the multiplication of whole numbers*
3. *On the addition of them, one to another*
4. *On the subtraction of lesser numbers from greater numbers*
5. *On the division of integral numbers*
6. *On the multiplication of integral numbers with fractions*
7. *On the addition and subtraction and division of numbers and fractions and the reduction of parts to a single part*
8. *On the buying and selling of commercial things (ratio & proportion)*
9. *On the barter of commercial things (rate)*
10. *On companies made among parties (percents)*
11. *On the alloying of money (mixture problems)*
12. *On the solutions of many problems (Fibonacci sequence)*
13. *On the rule of elchataym by which problems of false position are solved. (solving linear equations)*
14. *On the finding of square and cube roots, on binomials and their roots.*
15. *On the pertinent rules of geometric proportions*

2013

Became the curriculum staircase



Video on Common Core Math Standards

Sequence by the difficulty of the algorithms

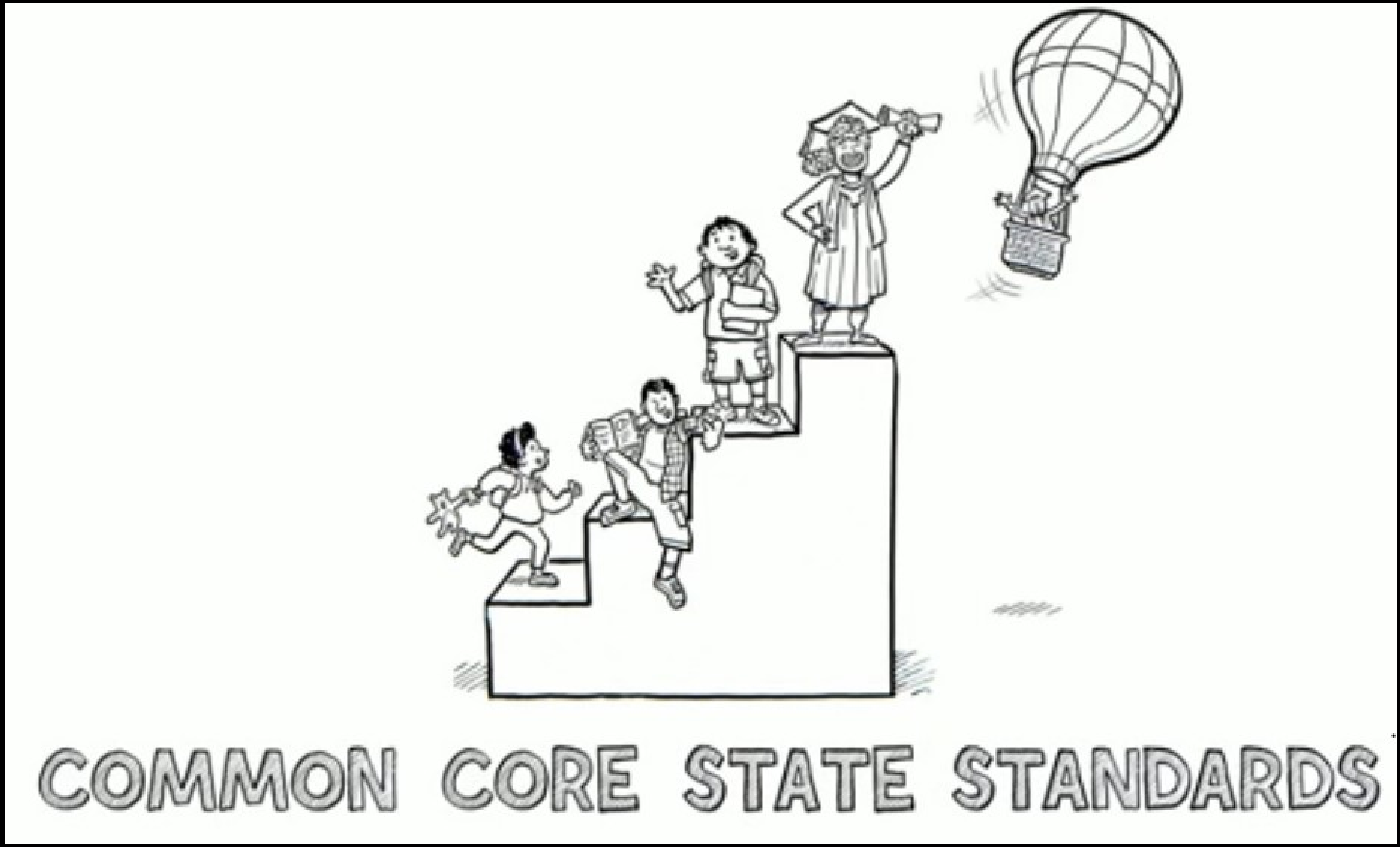
$$23\overline{)4381}$$

$$\begin{array}{r} 847 \\ \underline{\times\ 74} \end{array}$$

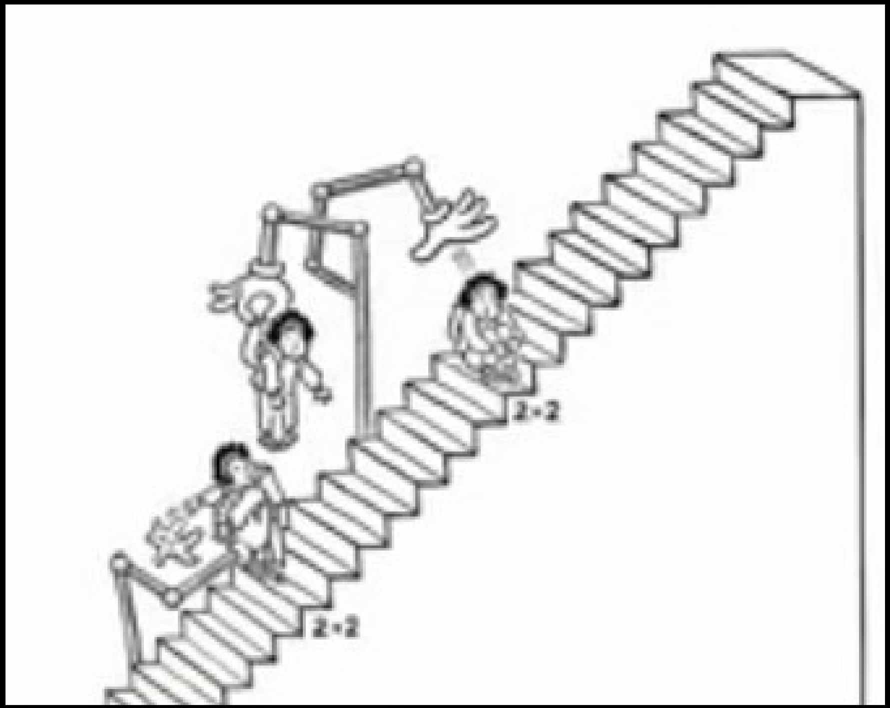
$$\begin{array}{r} 647 \\ \underline{-\ 49} \end{array}$$

$$\begin{array}{r} 5280 \\ \underline{+173} \end{array}$$

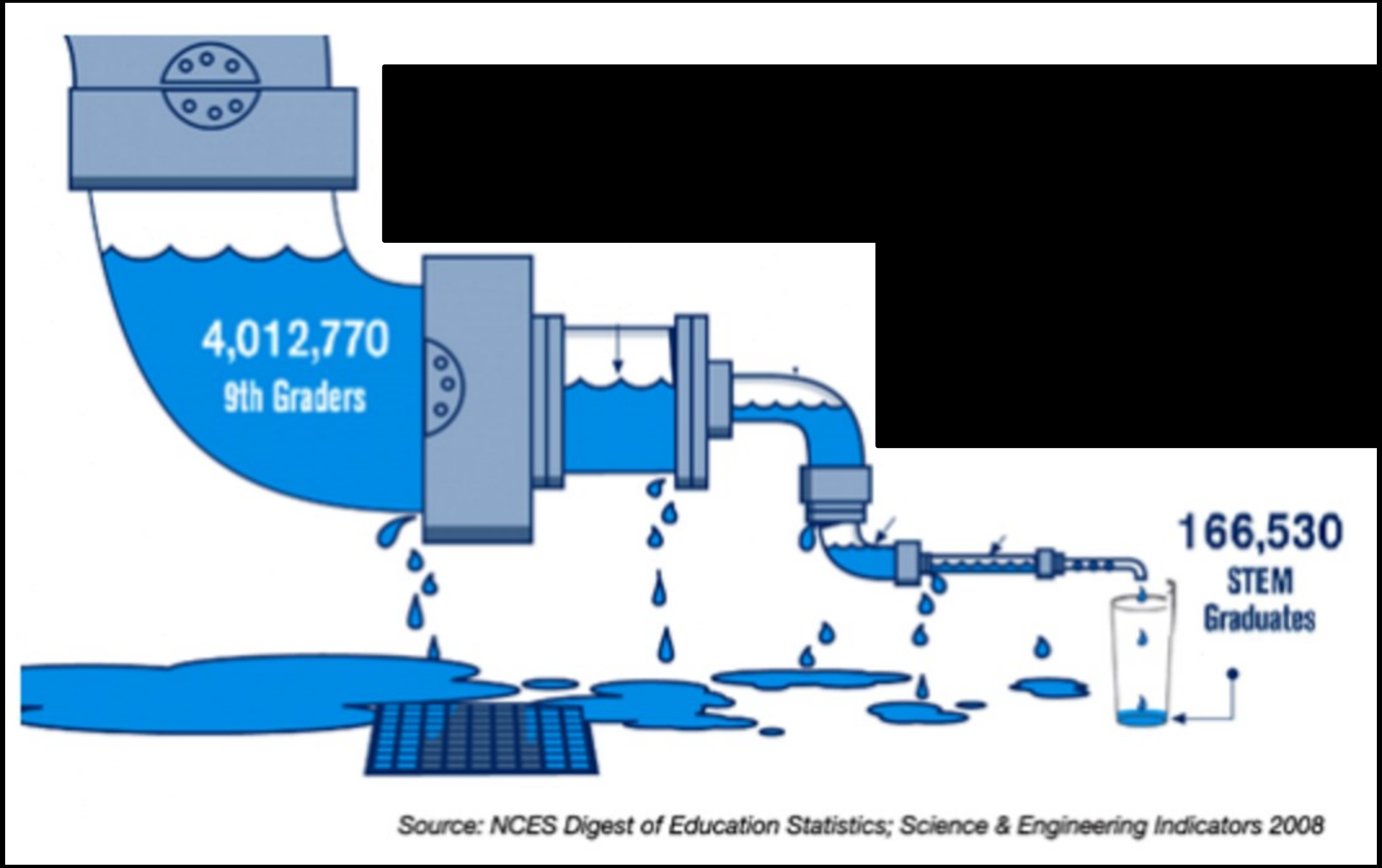
Every student still must climb today!



Yet, many “fall behind”
so many fail



Choking the STEM pipeline



$\underline{5280 + 1732 =}$

$$\begin{array}{r} 647 \\ \underline{\ {*}44} \end{array}$$

$6\frac{2}{3} - \frac{1}{8} =$

$\frac{5}{6} / \frac{-7}{12} =$

$\frac{438}{25} = 17\, r14$

$\sqrt[3]{64 + \sqrt{81}}$

$a^2 + b^2 = c^2$

What's worse

$A = \pi r^2$

$3x - 7 = 11$

$\frac{16}{9} = \frac{6}{x}$

$\frac{10}{7}x + 1 = \frac{3}{2}x - 8$

$2x^2 - 8x + 14$

$(15x^2 + 8x - 4)/(3x + 1)$

$\frac{-x}{x^2 - 6x + 5} + \frac{-x - 1}{x^2 - 10x + 25}$

$\frac{4}{6\sqrt{3}}$

$\sqrt[3]{6x - 4} = \sqrt[3]{5\mathrm{x} + 8}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

What's worse Leonardo's math problem solving is:

Obsolete

$\underline{5280 + 1732 =}$

$$\begin{array}{r} 647 \\ \underline{*44} \end{array}$$

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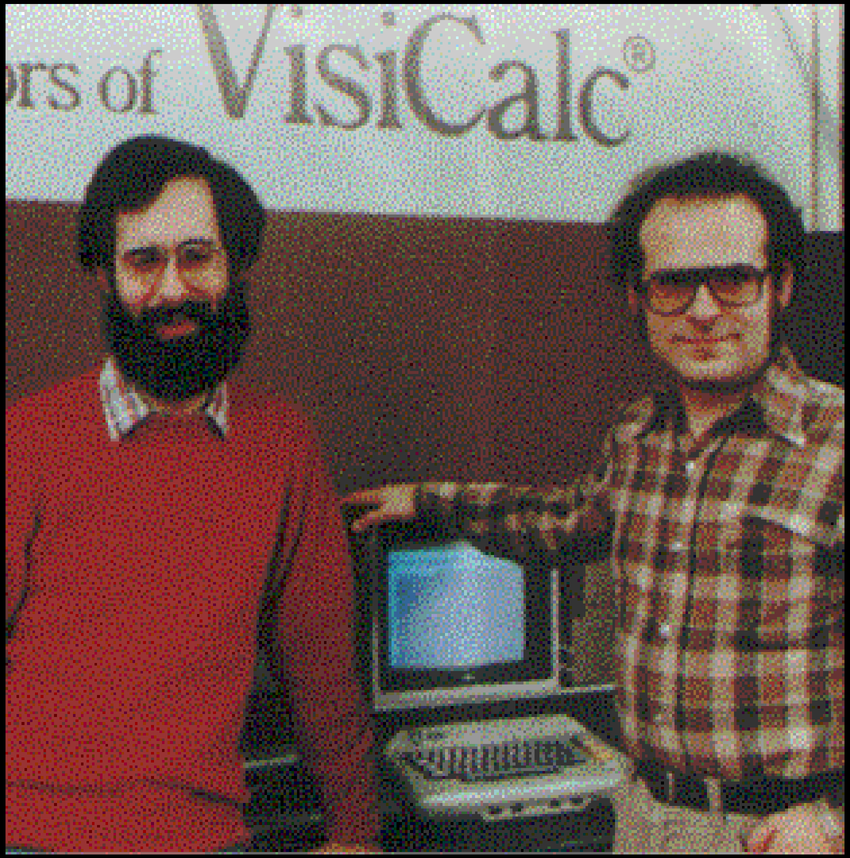
$\sqrt[3]{6x - 4} = \sqrt[3]{5\mathrm{x} + 8}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

For in 1979 a new technology reinvented business and STEM problem solving

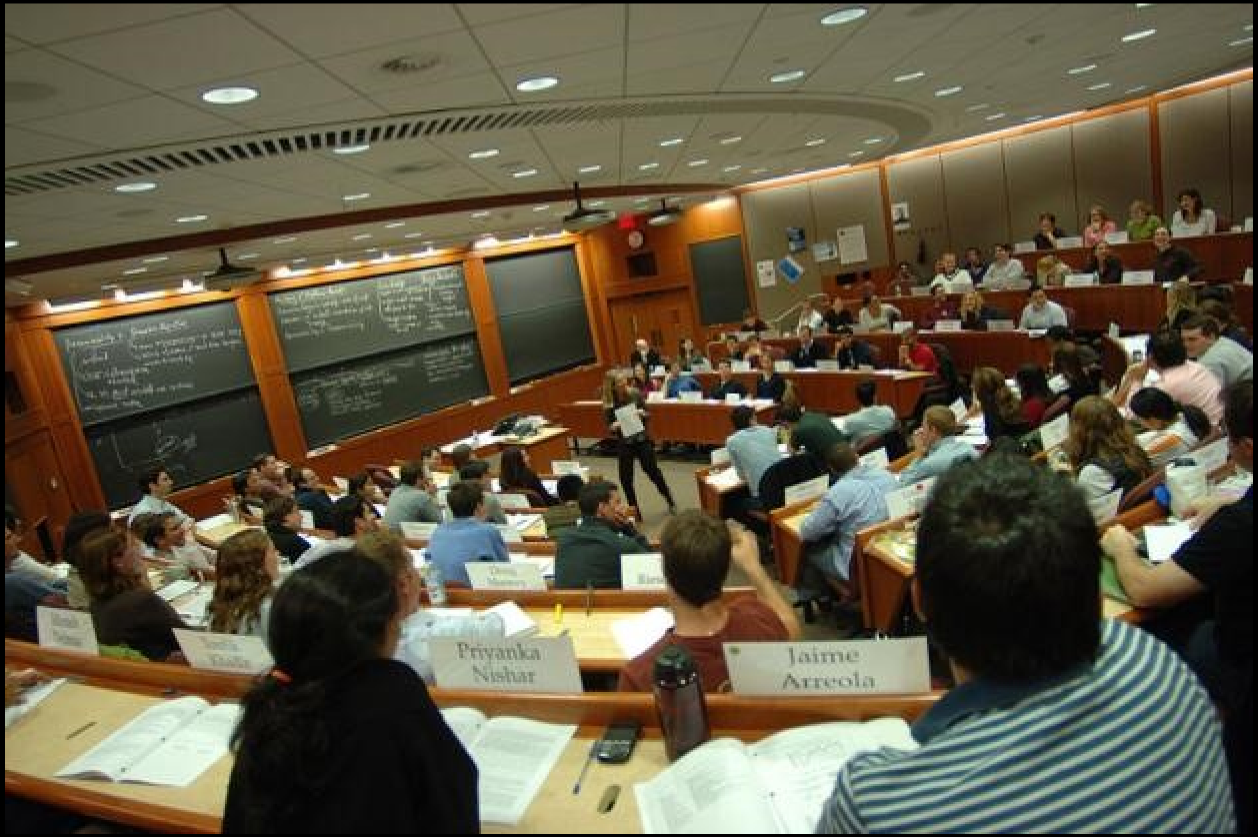
Dan Bricklin



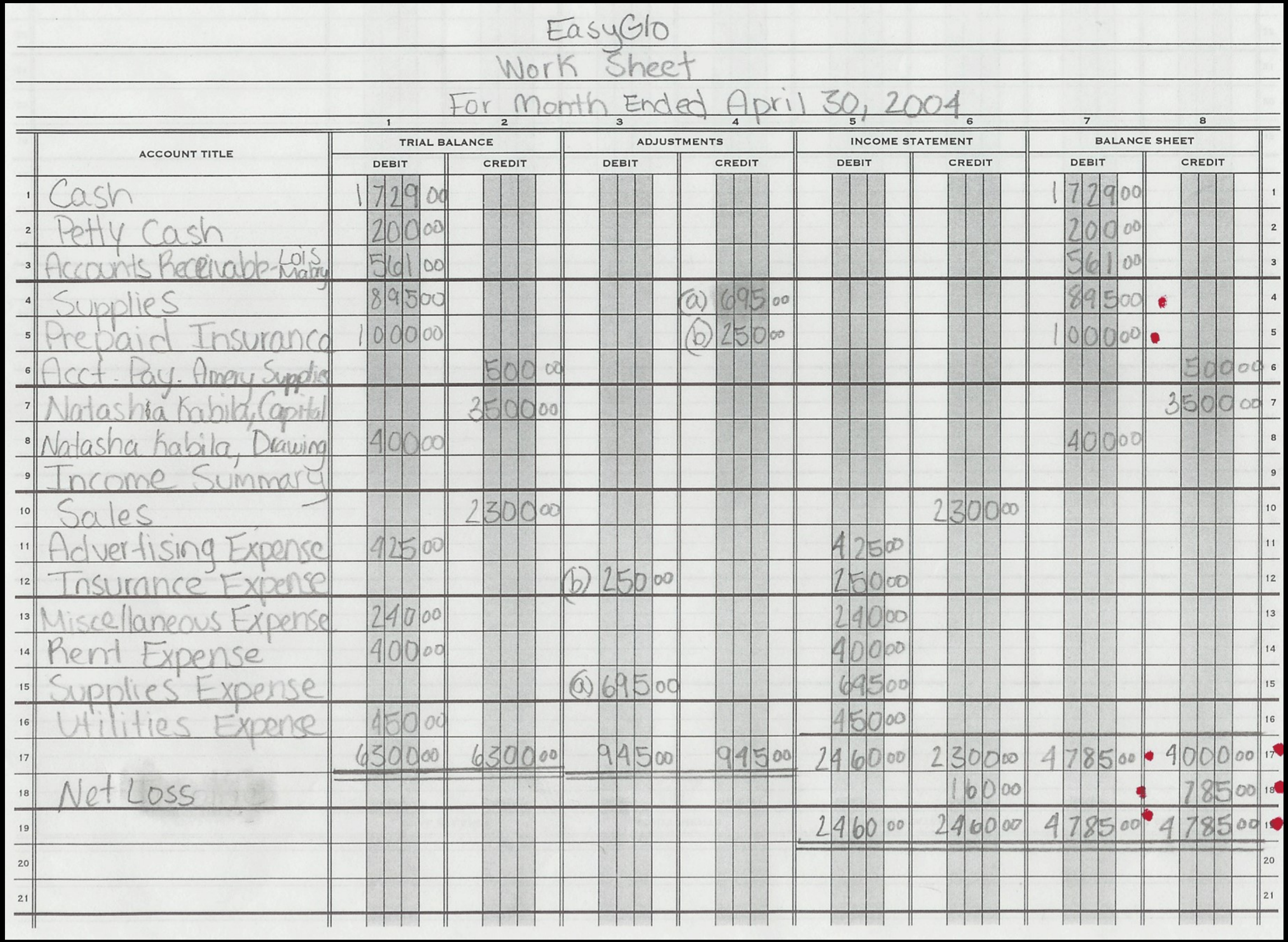
Dan Bricklin & Bob Frankston

A Harvard Business School student

Working on case studies



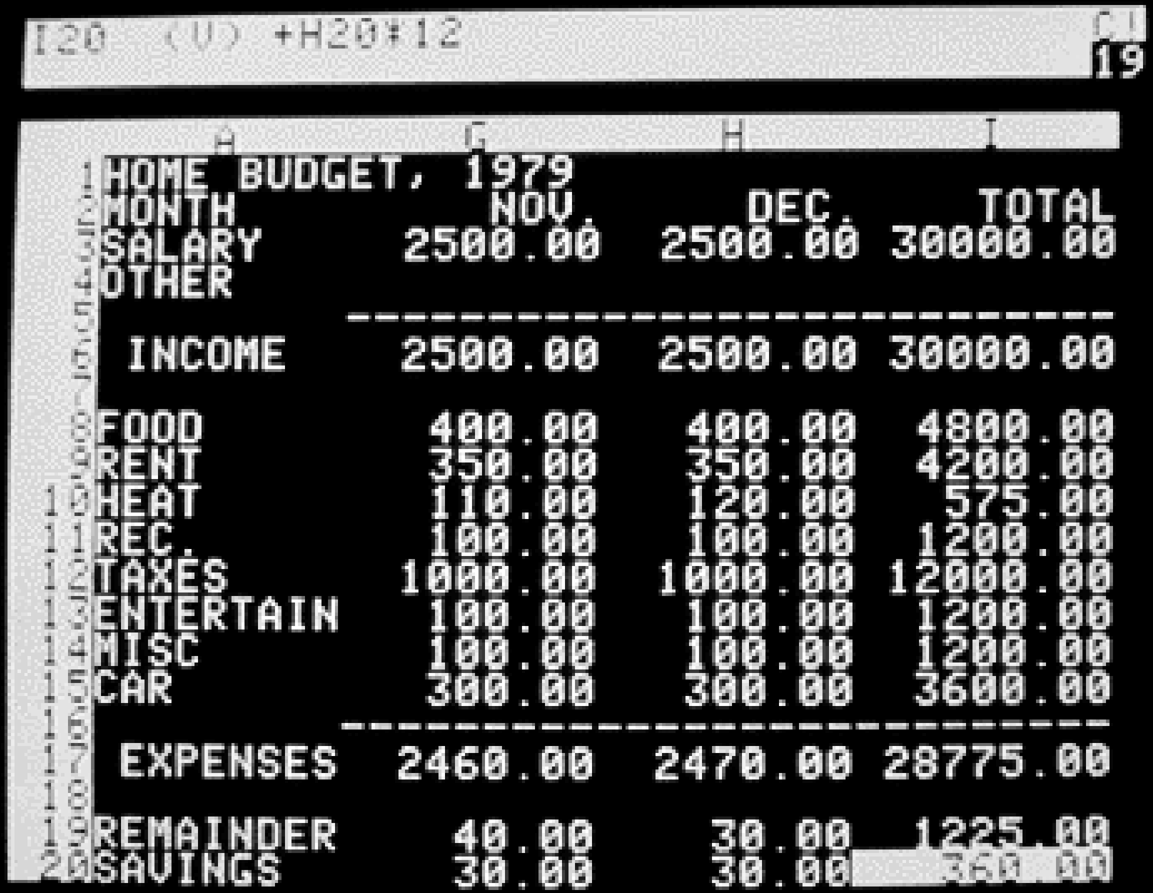
Wanted technology to enable him to ask “What if…”



EasyGlo
Work Sheet
For Month Ended April 30, 2004

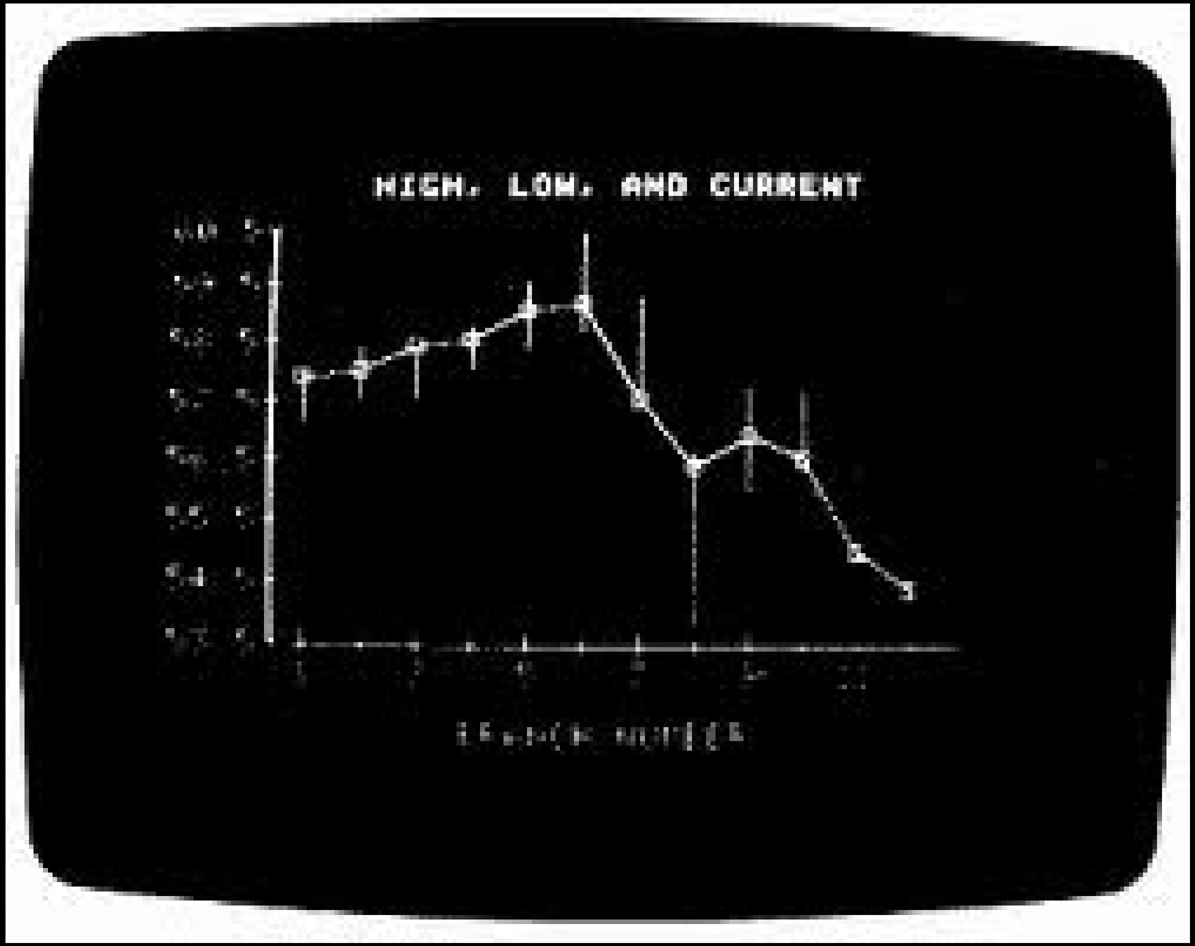
ACCOUNT TITLE	TRIAL BALANCE DEBIT (1)	TRIAL BALANCE CREDIT (2)	ADJUSTMENTS DEBIT (3)	ADJUSTMENTS CREDIT (4)	INCOME STATEMENT DEBIT (5)	INCOME STATEMENT CREDIT (6)	BALANCE SHEET DEBIT (7)	BALANCE SHEET CREDIT (8)
Cash	1729 00						1729 00	
Petty Cash	200 00						200 00	
Accounts Receivable-Lois Mary	561 00						561 00	
Supplies	895 00			(a) 695 00			895 00	
Prepaid Insurance	1000 00			(b) 250 00			1000 00	
Acct. Pay. Amery Supplies		500 00						500 00
Natashia Kabila, Capital		3500 00						3500 00
Natasha Kabila, Drawing	400 00						400 00	
Income Summary								
Sales		2300 00				2300 00		
Advertising Expense	425 00				425 00			
Insurance Expense			(b) 250 00		250 00			
Miscellaneous Expense	240 00				240 00			
Rent Expense	400 00				400 00			
Supplies Expense			(a) 695 00		695 00			
Utilities Expense	450 00				450 00			
	6300 00	6300 00	945 00	945 00	2460 00	2300 00	4785 00	4000 00
Net Loss						160 00		785 00
					2460 00	2460 00	4785 00	4785 00

So he and Bob Frankston invented the spreadsheet



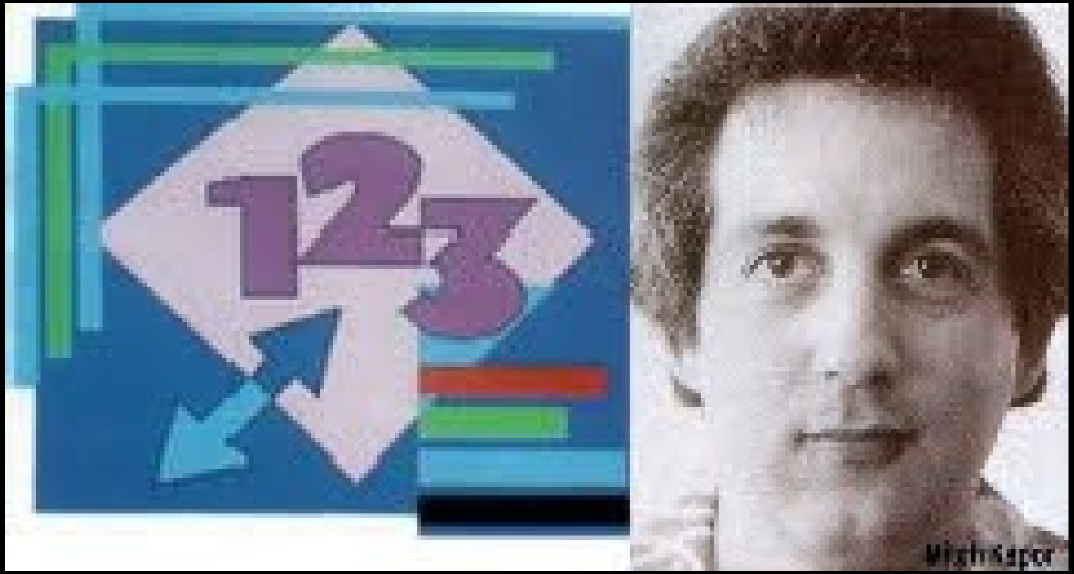
VisiCalc the Visible Calculator

Mitch Kapor added graphs



VisiPlot

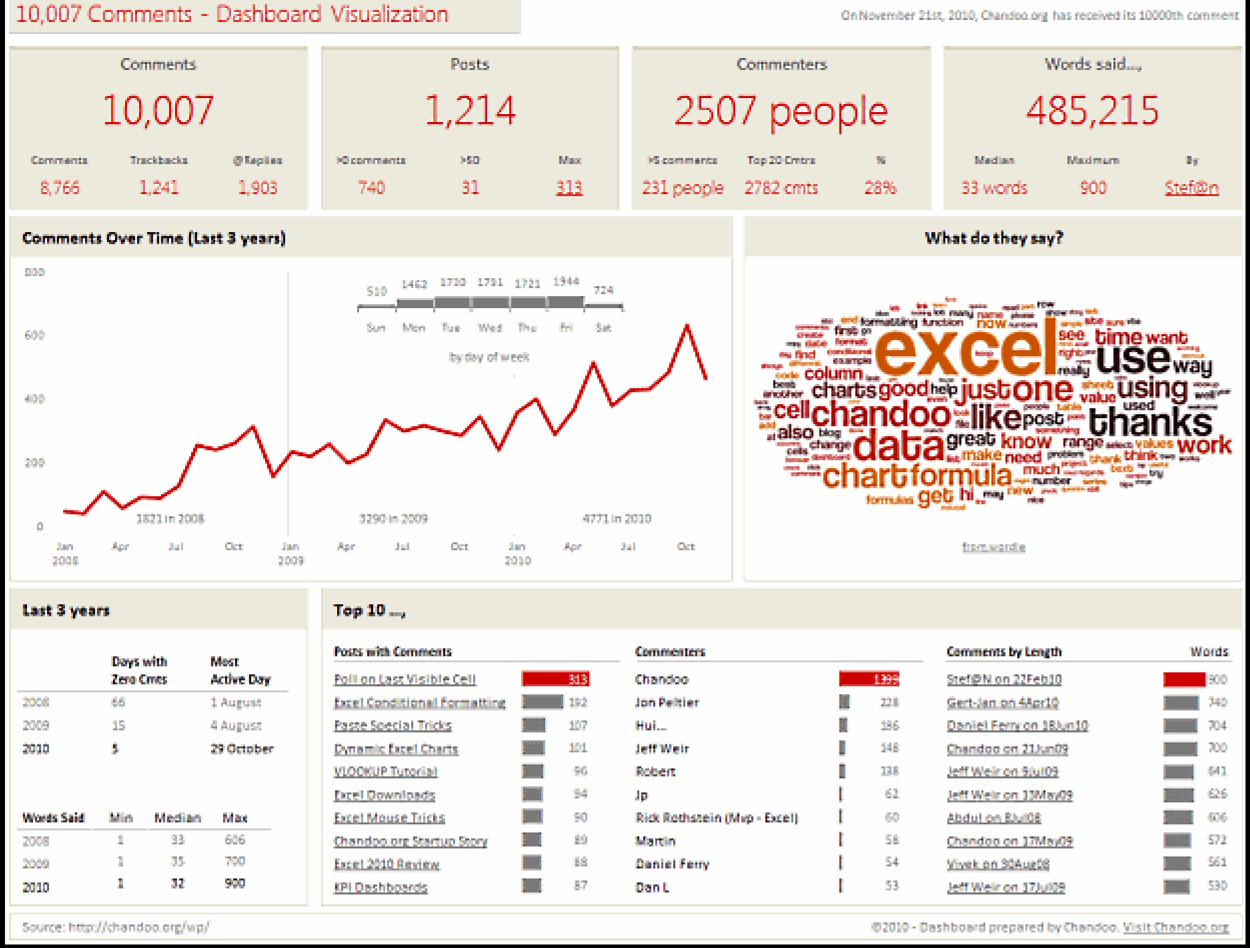
And then a database



Mitch and Lotus 123

Putting a PC on every business desk with…

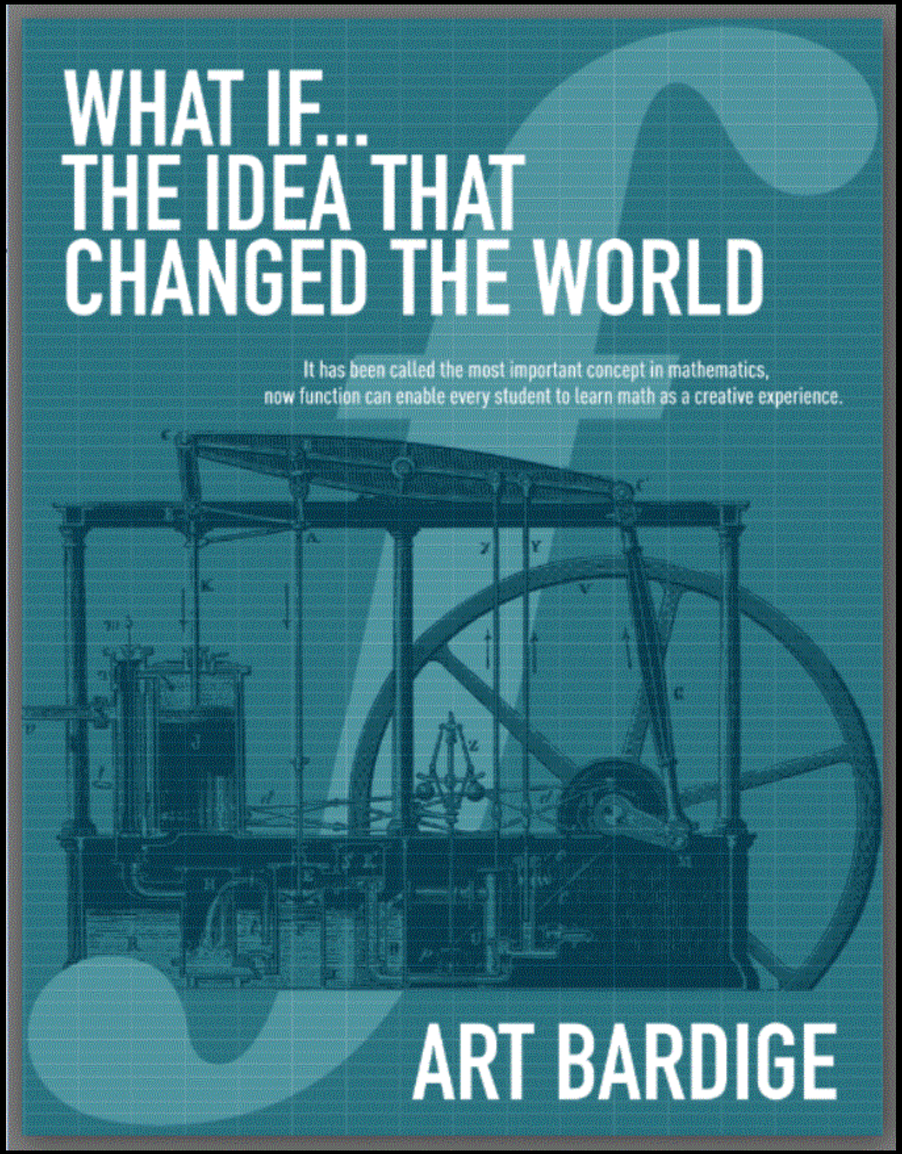
…a spreadsheet inside



To ask not "What is___?" but…

To ask not "What is____?" but… *"What if…"*

Spreadsheets are function machines

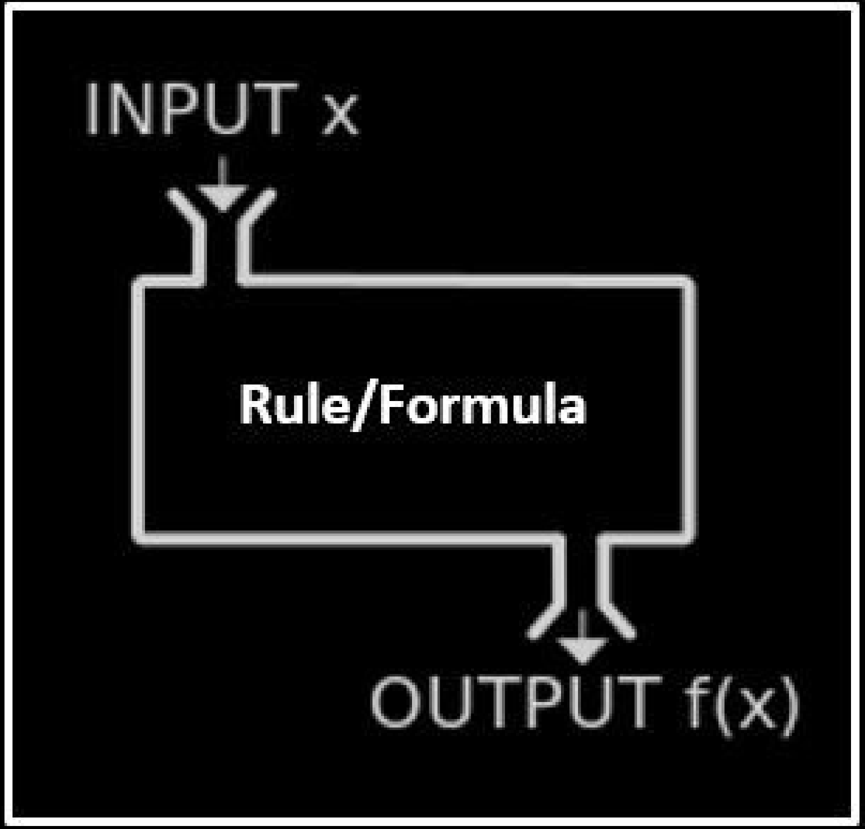


Function is…

"Perhaps **the most important concept of mathematics**… provides us with the means to study dependence and change."

Professor Peter Kronheimer, Director of Undergraduate Studies (2013-14)

Function

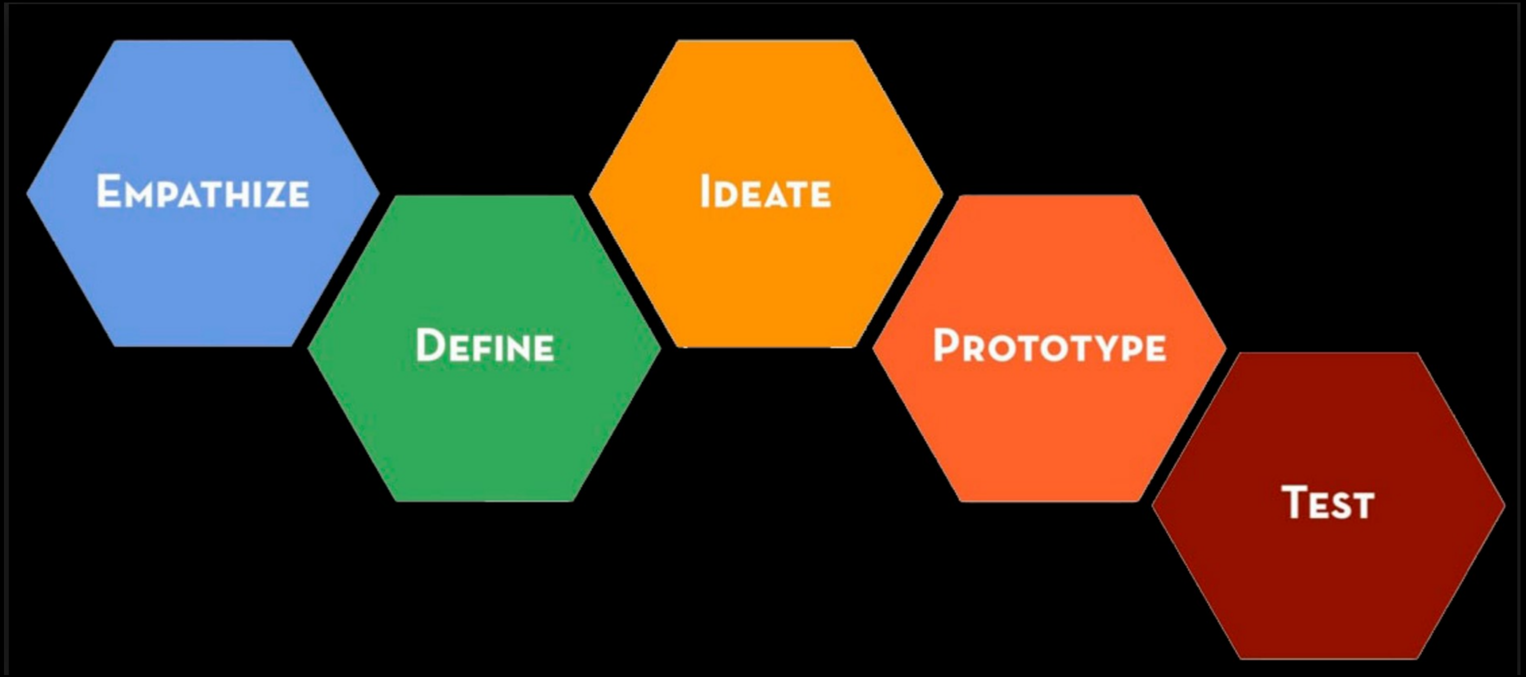


Science as experimentation

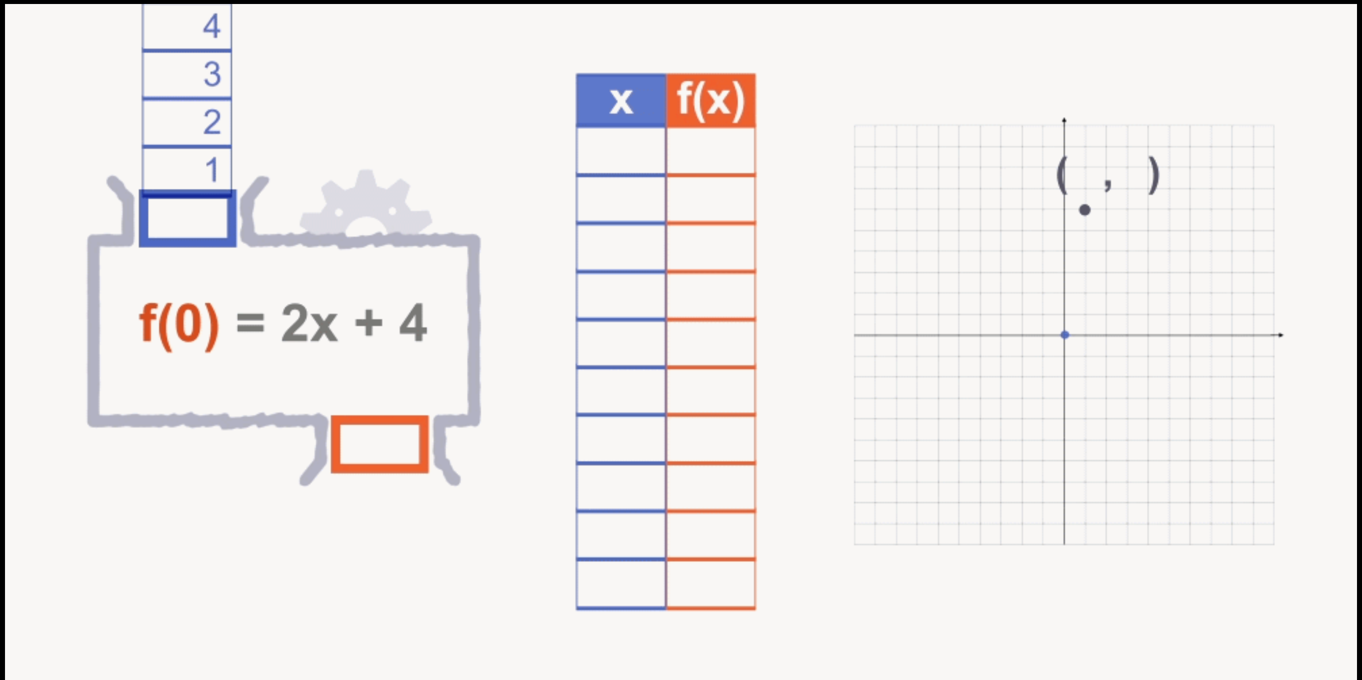
Mr. Wizard (Don Herbert) 1955

Technology as Coding

Engineering as Design Thinking

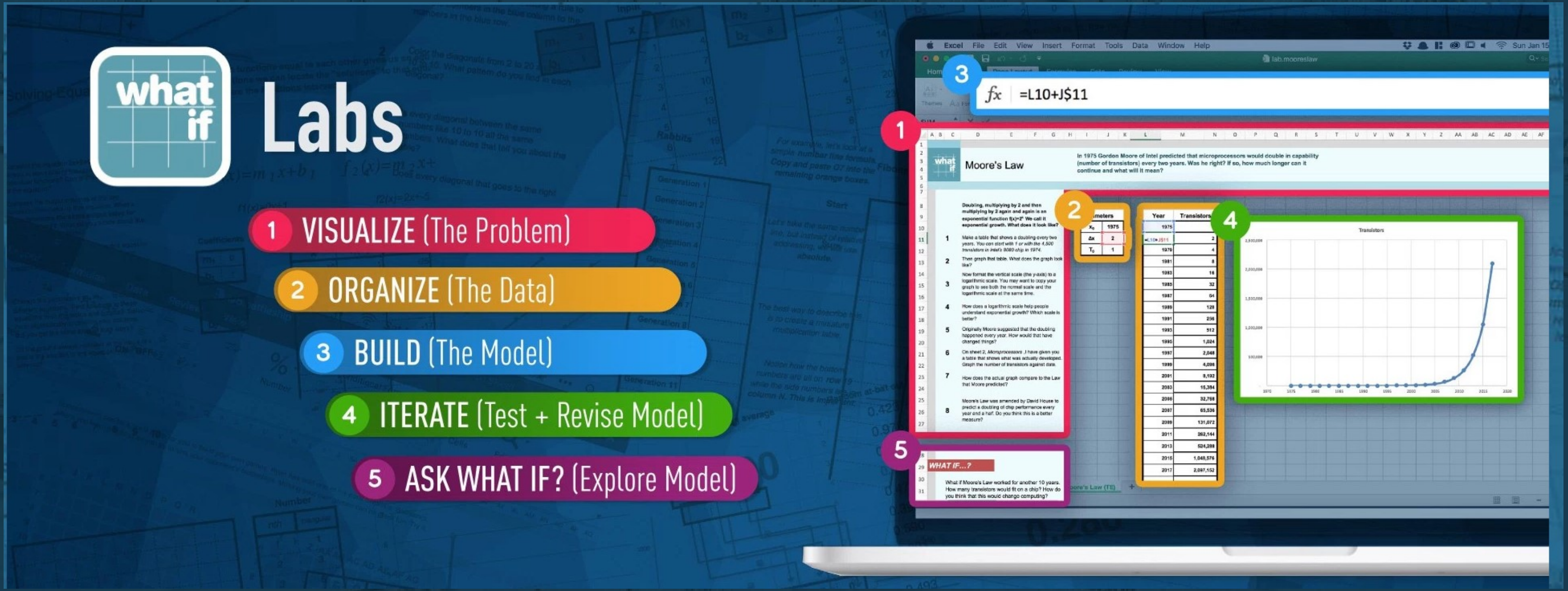


Math using digital tools

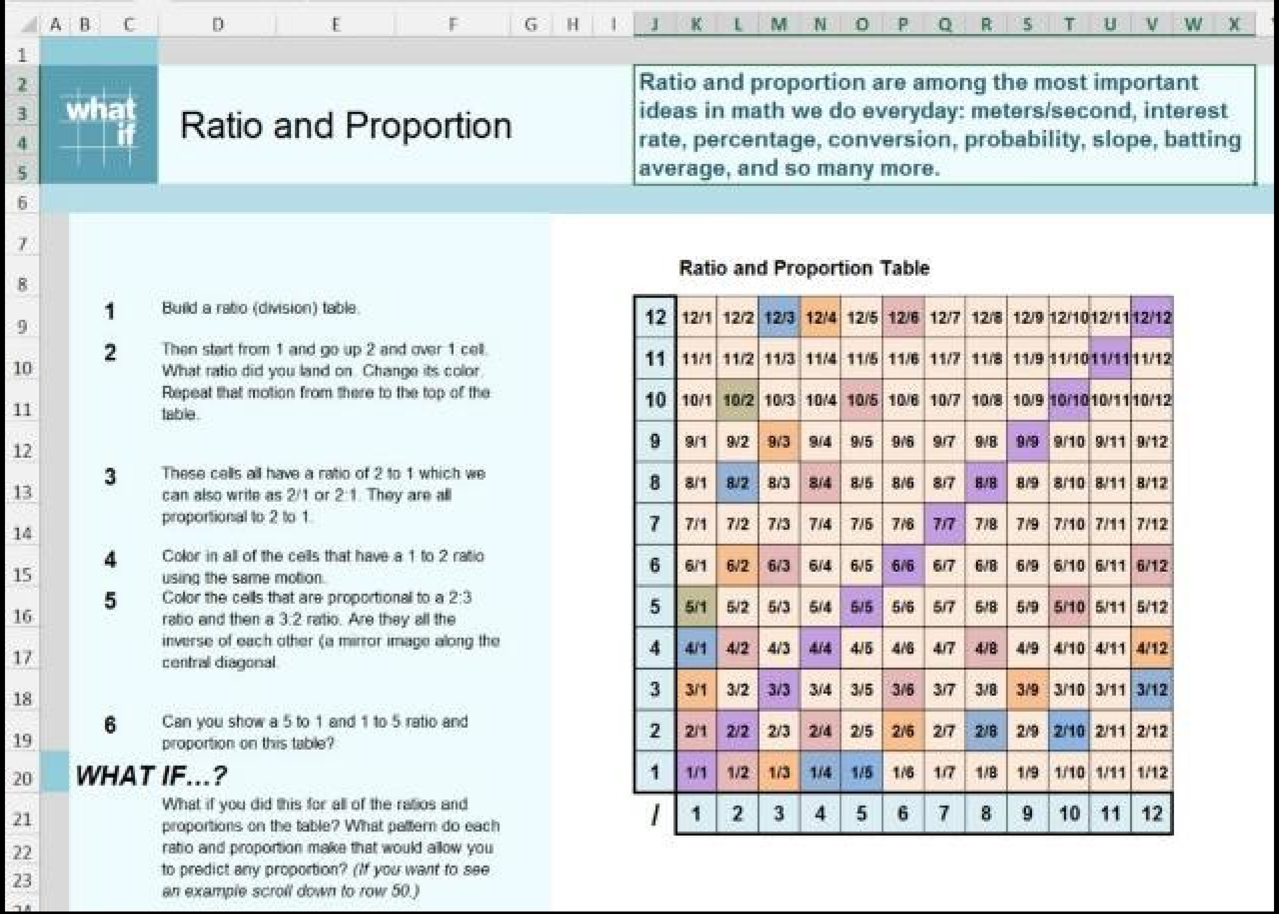


What if…

Students learned to problem solve using functional thinking on spreadsheets?



Visualize



Ratio and Proportion

Ratio and proportion are among the most important ideas in math we do everyday: meters/second, interest rate, percentage, conversion, probability, slope, batting average, and so many more.

1 Build a ratio (division) table.

2 Then start from 1 and go up 2 and over 1 cell. What ratio did you land on. Change its color. Repeat that motion from there to the top of the table.

3 These cells all have a ratio of 2 to 1 which we can also write as 2/1 or 2:1. They are all proportional to 2 to 1.

4 Color in all of the cells that have a 1 to 2 ratio using the same motion.

5 Color the cells that are proportional to a 2:3 ratio and then a 3:2 ratio. Are they all the inverse of each other (a mirror image along the central diagonal.

6 Can you show a 5 to 1 and 1 to 5 ratio and proportion on this table?

WHAT IF...?

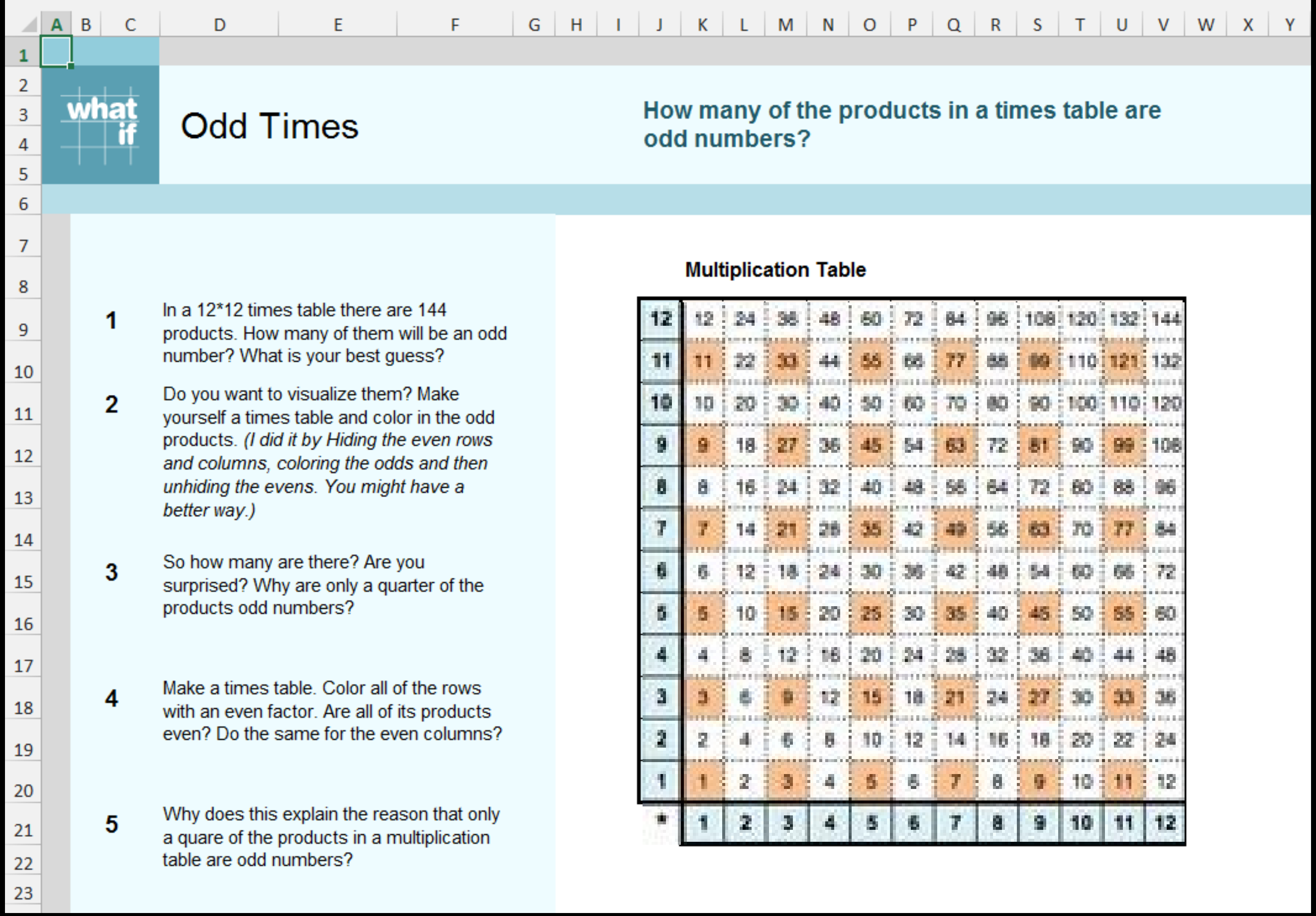
What if you did this for all of the ratios and proportions on the table? What pattern do each ratio and proportion make that would allow you to predict any proportion? *(If you want to see an example scroll down to row 50.)*

Ratio and Proportion Table

12	12/1	12/2	12/3	12/4	12/5	12/6	12/7	12/8	12/9	12/10	12/11	12/12
11	11/1	11/2	11/3	11/4	11/5	11/6	11/7	11/8	11/9	11/10	11/11	11/12
10	10/1	10/2	10/3	10/4	10/5	10/6	10/7	10/8	10/9	10/10	10/11	10/12
9	9/1	9/2	9/3	9/4	9/5	9/6	9/7	9/8	9/9	9/10	9/11	9/12
8	8/1	8/2	8/3	8/4	8/5	8/6	8/7	8/8	8/9	8/10	8/11	8/12
7	7/1	7/2	7/3	7/4	7/5	7/6	7/7	7/8	7/9	7/10	7/11	7/12
6	6/1	6/2	6/3	6/4	6/5	6/6	6/7	6/8	6/9	6/10	6/11	6/12
5	5/1	5/2	5/3	5/4	5/5	5/6	5/7	5/8	5/9	5/10	5/11	5/12
4	4/1	4/2	4/3	4/4	4/5	4/6	4/7	4/8	4/9	4/10	4/11	4/12
3	3/1	3/2	3/3	3/4	3/5	3/6	3/7	3/8	3/9	3/10	3/11	3/12
2	2/1	2/2	2/3	2/4	2/5	2/6	2/7	2/8	2/9	2/10	2/11	2/12
1	1/1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/10	1/11	1/12
/	1	2	3	4	5	6	7	8	9	10	11	12

Ratio & Proportion (Grades 4-6)

Organize



what if

Odd Times

How many of the products in a times table are odd numbers?

1 In a 12*12 times table there are 144 products. How many of them will be an odd number? What is your best guess?

2 Do you want to visualize them? Make yourself a times table and color in the odd products. *(I did it by Hiding the even rows and columns, coloring the odds and then unhiding the evens. You might have a better way.)*

3 So how many are there? Are you surprised? Why are only a quarter of the products odd numbers?

4 Make a times table. Color all of the rows with an even factor. Are all of its products even? Do the same for the even columns?

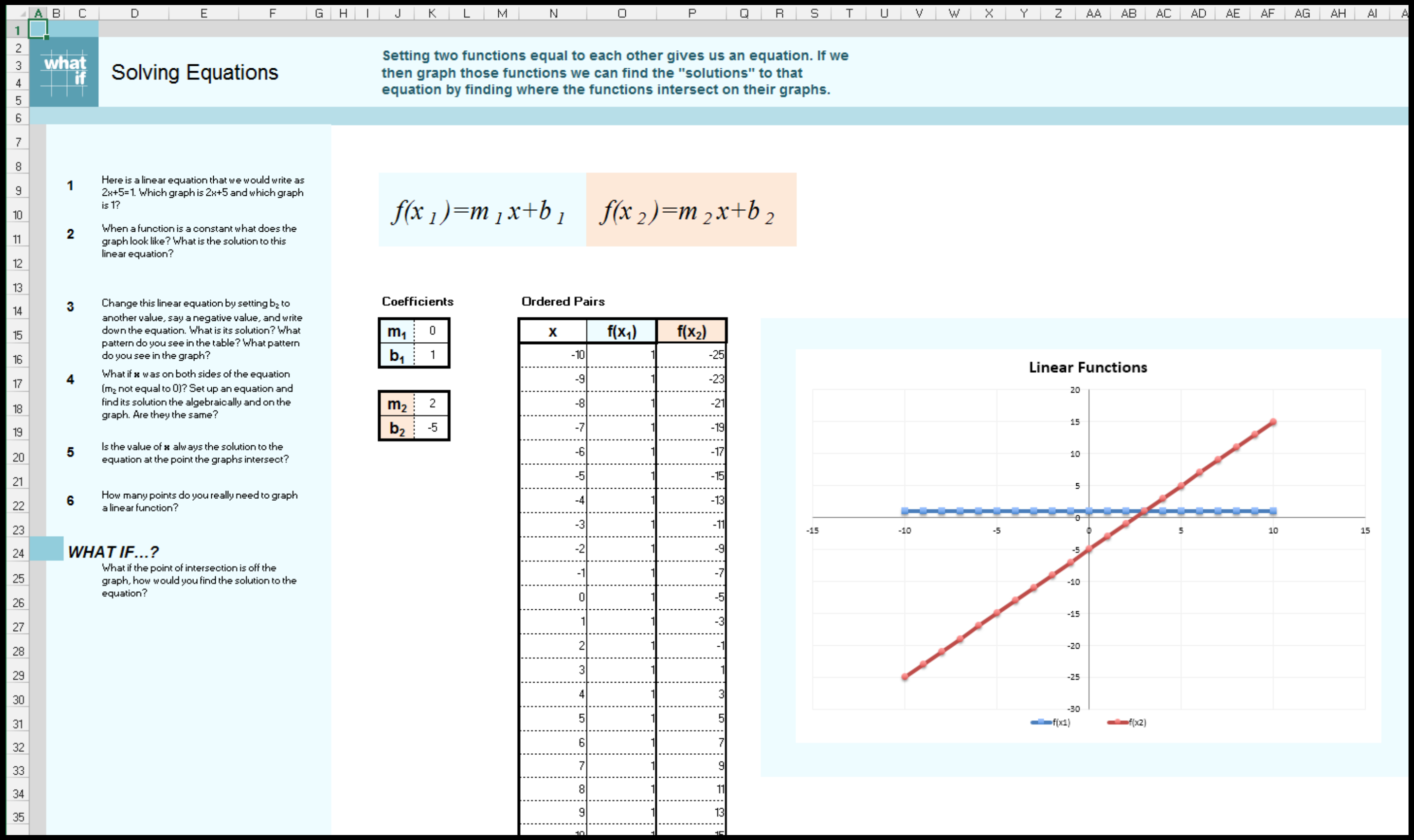
5 Why does this explain the reason that only a quare of the products in a multiplication table are odd numbers?

Multiplication Table

12	12	24	36	48	60	72	84	96	108	120	132	144
11	11	22	33	44	55	66	77	88	99	110	121	132
10	10	20	30	40	50	60	70	80	90	100	110	120
9	9	18	27	36	45	54	63	72	81	90	99	108
8	8	16	24	32	40	48	56	64	72	80	88	96
7	7	14	21	28	35	42	49	56	63	70	77	84
6	6	12	18	24	30	36	42	48	54	60	66	72
5	5	10	15	20	25	30	35	40	45	50	55	60
4	4	8	12	16	20	24	28	32	36	40	44	48
3	3	6	9	12	15	18	21	24	27	30	33	36
2	2	4	6	8	10	12	14	16	18	20	22	24
1	1	2	3	4	5	6	7	8	9	10	11	12
*	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

Odd Times (Grades 2-7)

Build Models



Solving Equations (Grades 6-12)

Iterate

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X

what if

Build a Times Table

Let's build a multiplication table in two steps and in the process learn about absolute vs. relative addressing.

1 To build a times table we need a vertical axis like our horizontal axis. Fill in the vertical axis for 1 to 12 going up.

2 You could type in the blue cell =**2*2**. What would happen if you copied that formula into all of the cells in the table?

3 What if you made the product from the addresses of the axes numbers? = **J18*N21** for example? What would happen if you copied that formula into all of the cells in a row or in the table?

4 Describe what went wrong? Do you have a hypothesis?

5 **Relative addressing** means that when you copy cells the addresses automatically move to form the same pattern.

6 **Absolute addressing** means that the cell addresses are fixed and they do not move when you copy cells from one location to another. Just put a $ in front of the address.

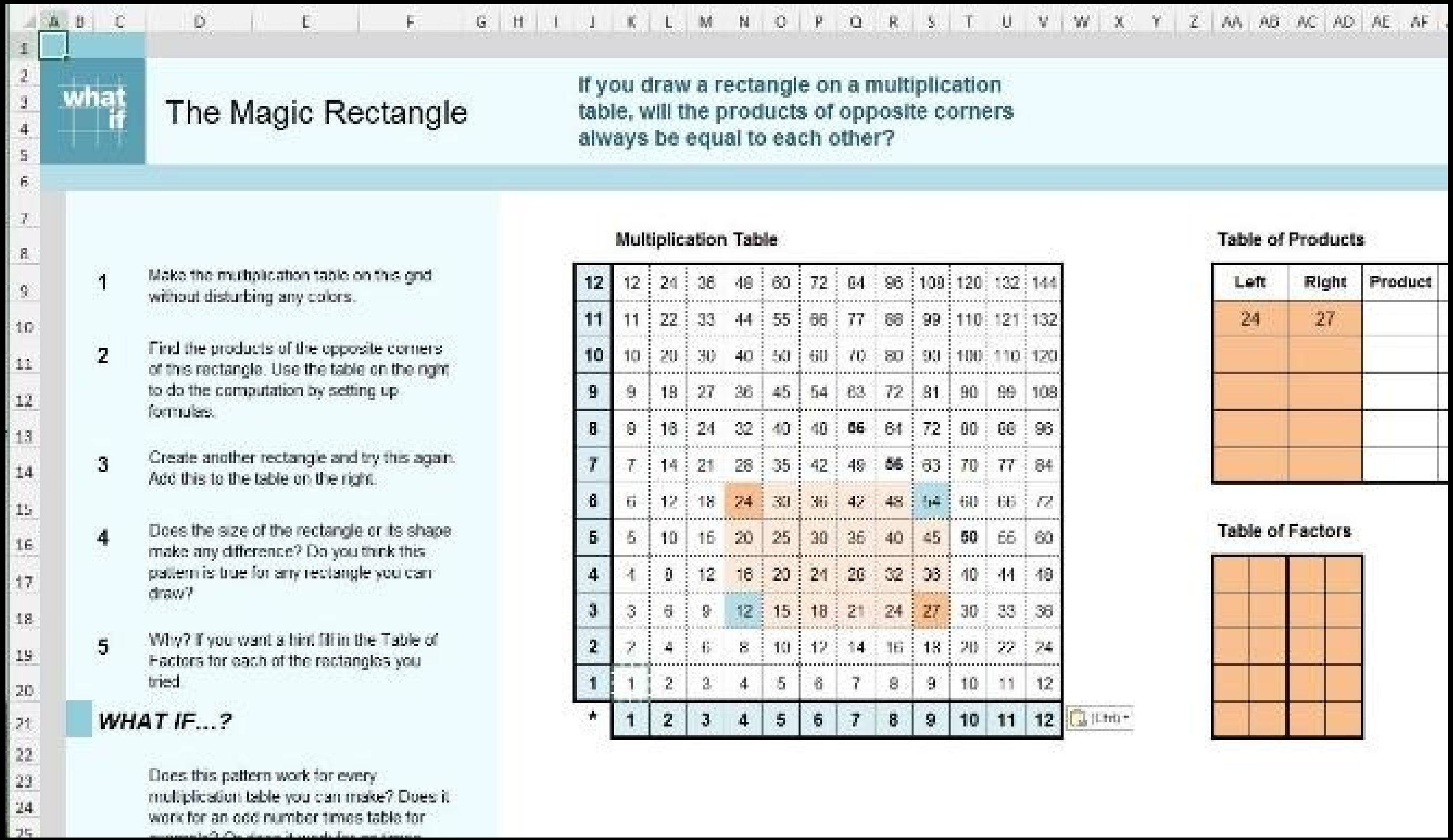
WHAT IF…?

Multiplication Table

12	12	24	36	48	60	72	84	96	108	120	132	144
11	11	22	33	44	55	66	77	88	99	110	121	132
10	10	20	30	40	50	60	70	80	90	100	110	120
9	9	18	27	36	45	54	63	72	81	90	99	108
8	8	16	24	32	40	48	56	64	72	80	88	96
7	7	14	21	28	35	42	49	56	63	70	77	84
6	6	12	18	24	30	36	42	48	54	60	66	72
5	5	10	15	20	25	30	35	40	45	50	55	60
4	4	8	12	16	20	24	28	32	36	40	44	48
3	3	6	9	12	15	18	21	24	27	30	33	36
2	2	4	6	8	10	12	14	16	18	20	22	24
1	1	2	3	4	5	6	7	8	9	10	11	12
*	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

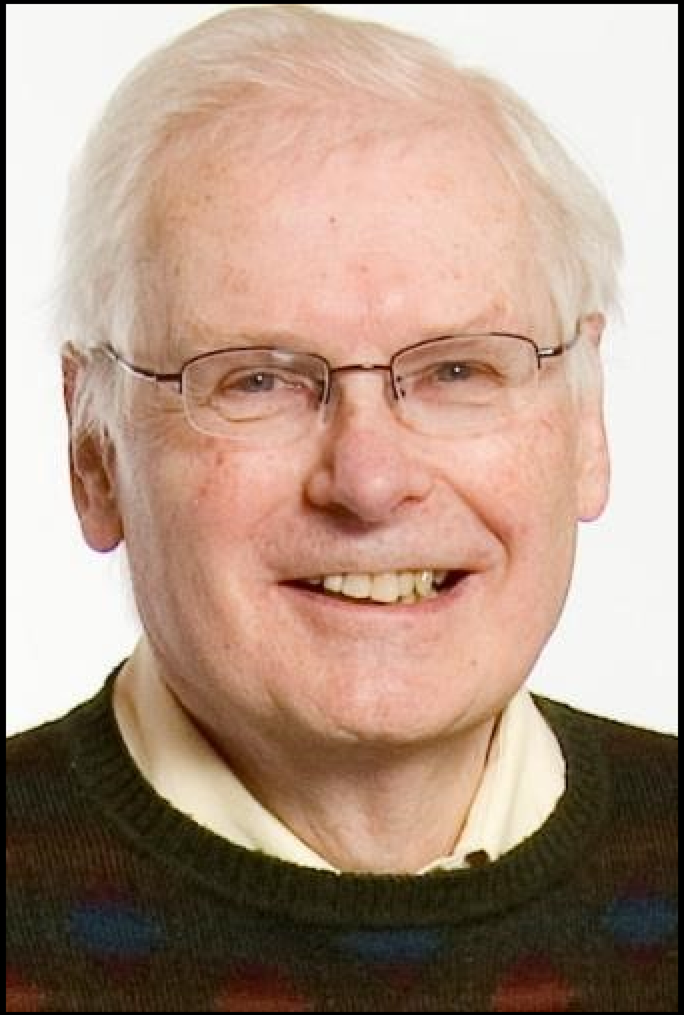
Build a Times Table (Grades 2-3)

Ask “What if…”

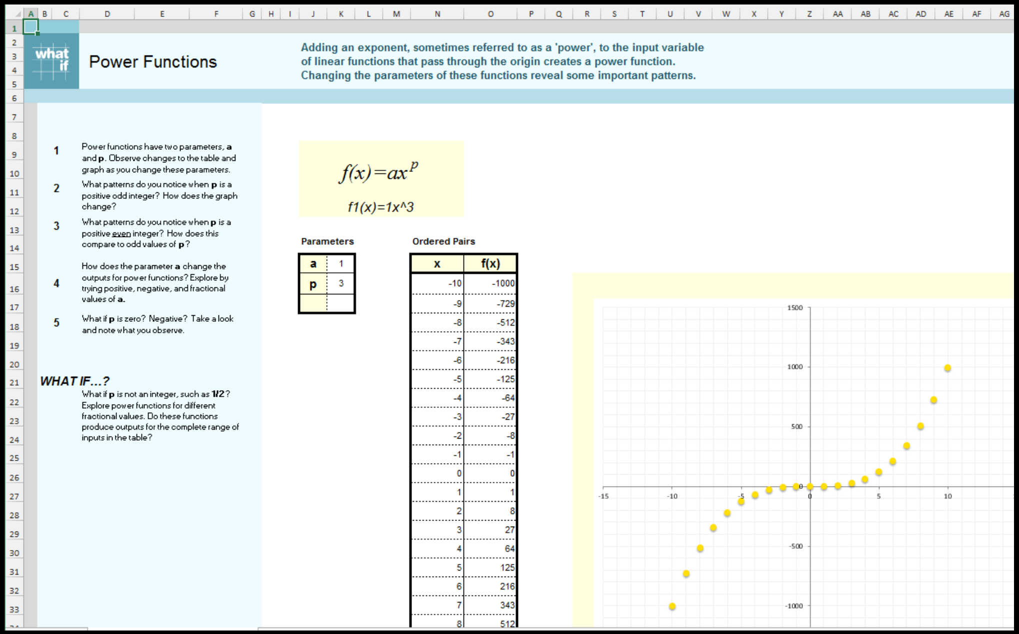
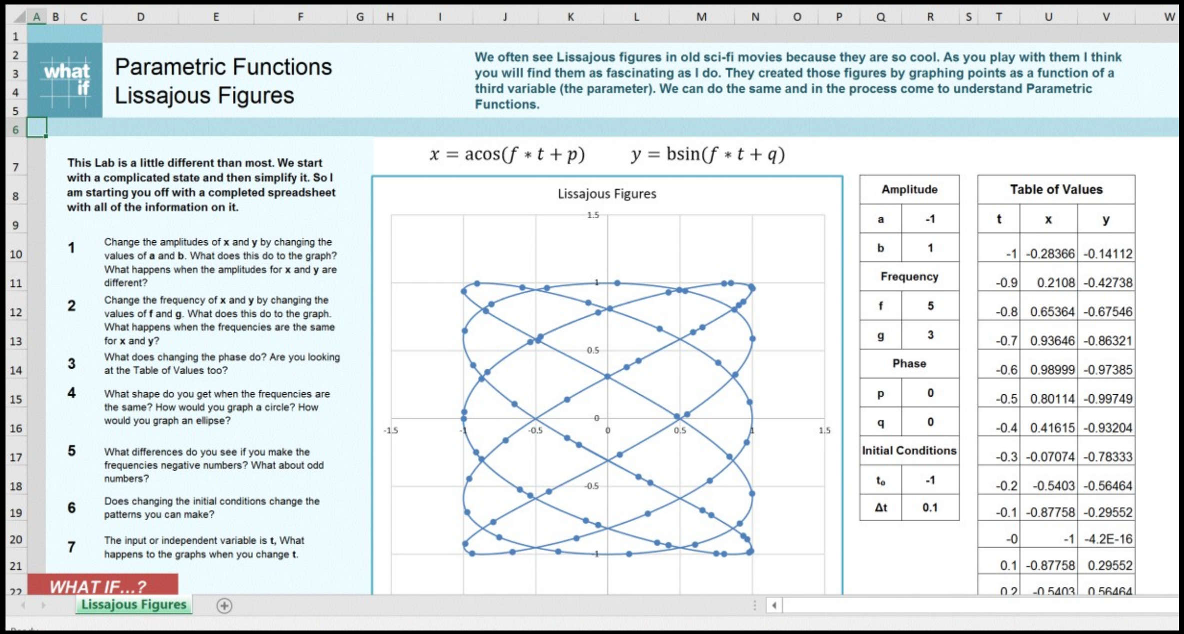
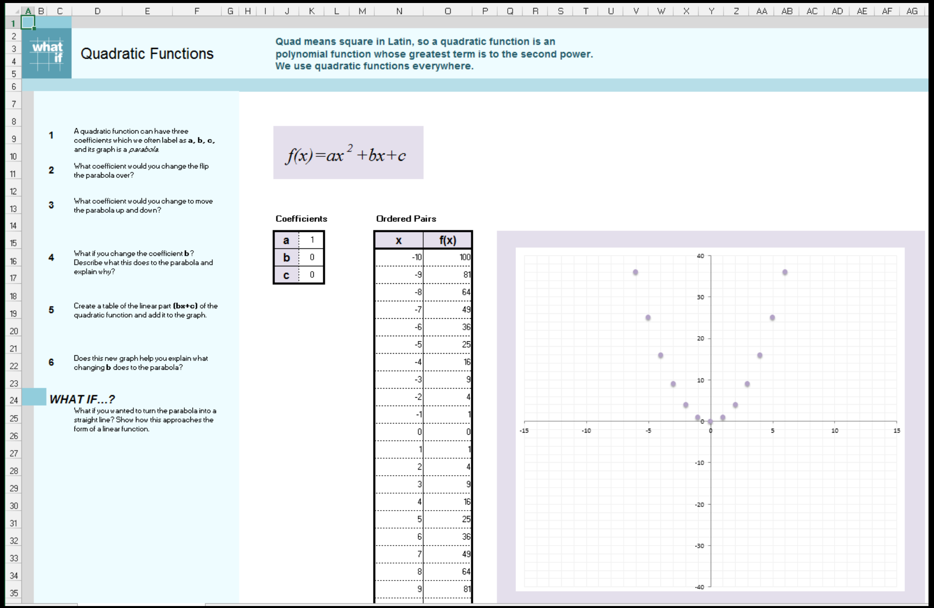
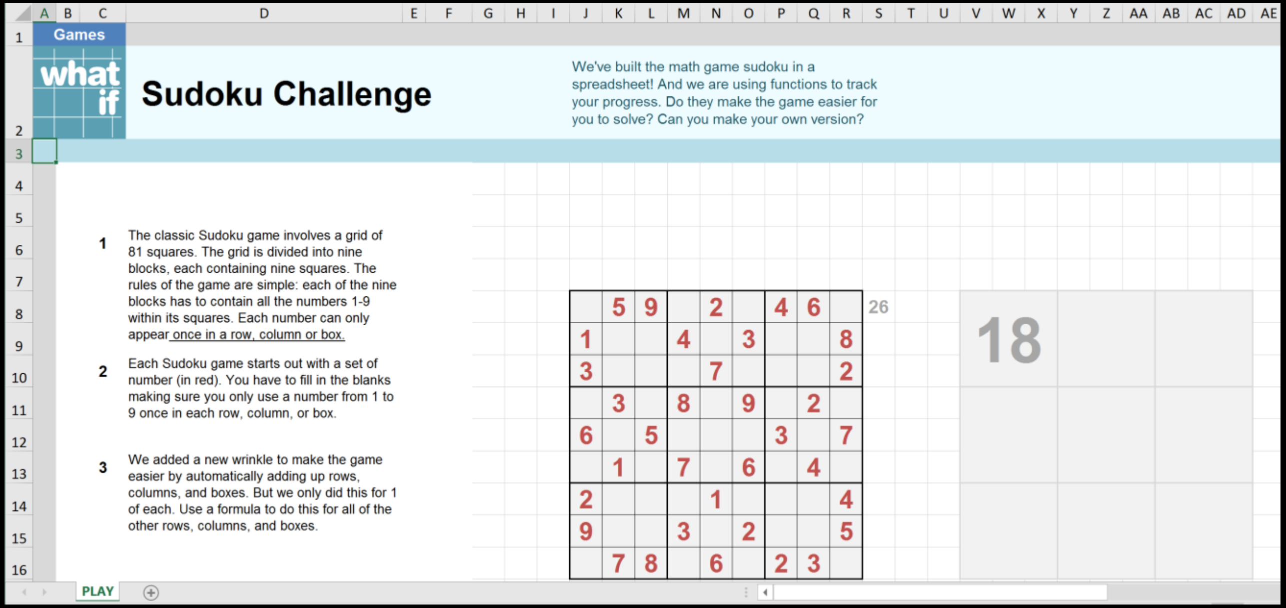


The Magic Rectangle (Grades 2-6)

They would learn to see mathematics as "The Science of Patterns"

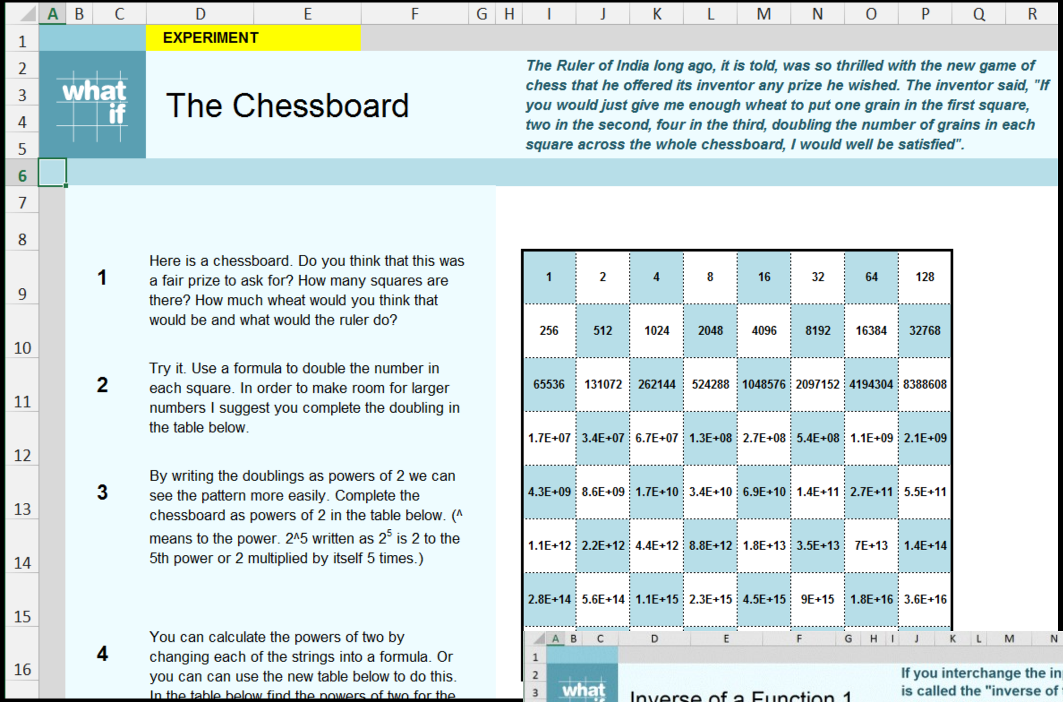


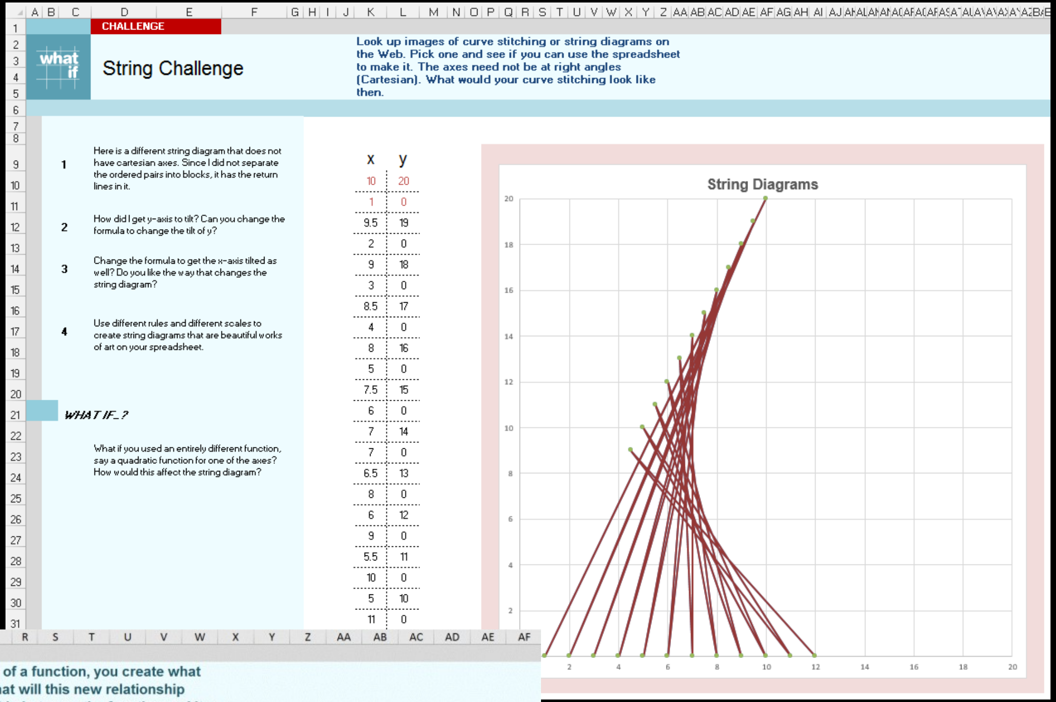
Lynn Arthur Steen (1941-2015)

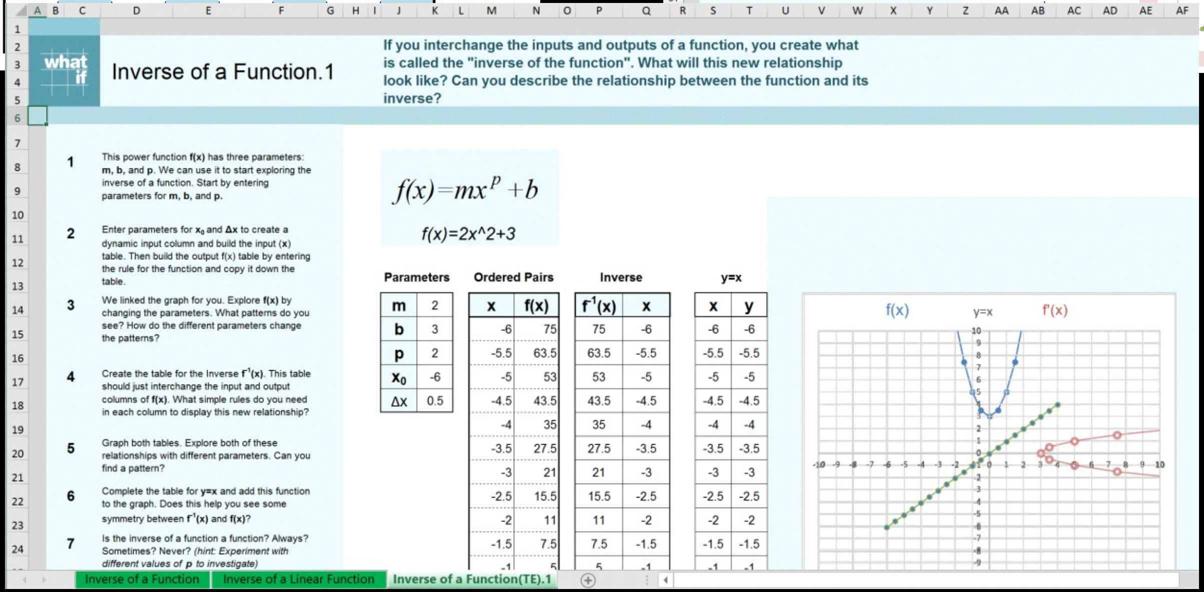

what if
Power Functions
Adding an exponent, sometimes referred to as a 'power', to the input variable of linear functions that pass through the origin creates a power function. Changing the parameters of these functions reveal some important patterns.
1 Power functions have two parameters, a and p. Observe changes to the table and graph as you change these parameters.
2 What patterns do you notice when p is a positive odd integer? How does the graph change?
3 What patterns do you notice when p is a positive even integer? How does this compare to odd values of p?
4 How does the parameter a change the outputs for power functions? Explore by trying positive, negative, and fractional values of a.
5 What if p is zero? Negative? Take a look and note what you observe.
WHAT IF...?
What if p is not an integer, such as 1/2? Explore power functions for different fractional values. Do these functions produce outputs for the complete range of inputs in the table?
f(x)=ax^p
f1(x)=1x^3
Parameters
a 1
p 3
Ordered Pairs
x f(x)

what if
Parametric Functions
Lissajous Figures
We often see Lissajous figures in old sci-fi movies because they are so cool. As you play with them I think you will find them as fascinating as I do. They created those figures by graphing points as a function of a third variable (the parameter). We can do the same and in the process come to understand Parametric Functions.
This Lab is a little different than most. We start with a complicated state and then simplify it. So I am starting you off with a completed spreadsheet with all of the information on it.
1 Change the amplitudes of x and y by changing the values of a and b. What does this do to the graph? What happens when the amplitudes for x and y are different?
2 Change the frequency of x and y by changing the values of f and g. What does this do the graph. What happens when the frequencies are the same for x and y?
3 What does changing the phase do? Are you looking at the Table of Values too?
4 What shape do you get when the frequencies are the same? How would you graph a circle? How would you graph an ellipse?
5 What differences do you see if you make the frequencies negative numbers? What about odd numbers?
6 Does changing the initial conditions change the patterns you can make?
7 The input or independent variable is t, What happens to the graphs when you change t.
WHAT IF...?
x = acos(f ∗ t + p)   y = bsin(f ∗ t + q)
Lissajous Figures
Amplitude
a -1
b 1
Frequency
f 5
g 3
Phase
p 0
q 0
Initial Conditions
t₀ -1
Δt 0.1
Table of Values
t x y
Lissajous Figures

what if
Quadratic Functions
Quad means square in Latin, so a quadratic function is an polynomial function whose greatest term is to the second power. We use quadratic functions everywhere.
1 A quadratic function can have three coefficients which we often label as a, b, c, and its graph is a parabola.
2 What coefficient would you change the flip the parabola over?
3 What coefficient would you change to move the parabola up and down?
4 What if you change the coefficient b? Describe what this does to the parabola and explain why?
5 Create a table of the linear part (bx+c) of the quadratic function and add it to the graph.
6 Does this new graph help you explain what changing b does to the parabola?
WHAT IF...?
What if you wanted to turn the parabola into a straight line? Show how this approaches the form of a linear function.
f(x)=ax^2+bx+c
Coefficients
a 1
b 0
c 0
Ordered Pairs
x f(x)

Games
what if
Sudoku Challenge
We've built the math game sudoku in a spreadsheet! And we are using functions to track your progress. Do they make the game easier for you to solve? Can you make your own version?
1 The classic Sudoku game involves a grid of 81 squares. The grid is divided into nine blocks, each containing nine squares. The rules of the game are simple: each of the nine blocks has to contain all the numbers 1-9 within its squares. Each number can only appear once in a row, column or box.
2 Each Sudoku game starts out with a set of number (in red). You have to fill in the blanks making sure you only use a number from 1 to 9 once in each row, column, or box.
3 We added a new wrinkle to make the game easier by automatically adding up rows, columns, and boxes. But we only did this for 1 of each. Use a formula to do this for all of the other rows, columns, and boxes.
26
18
PLAY

Think-out-of-the-box

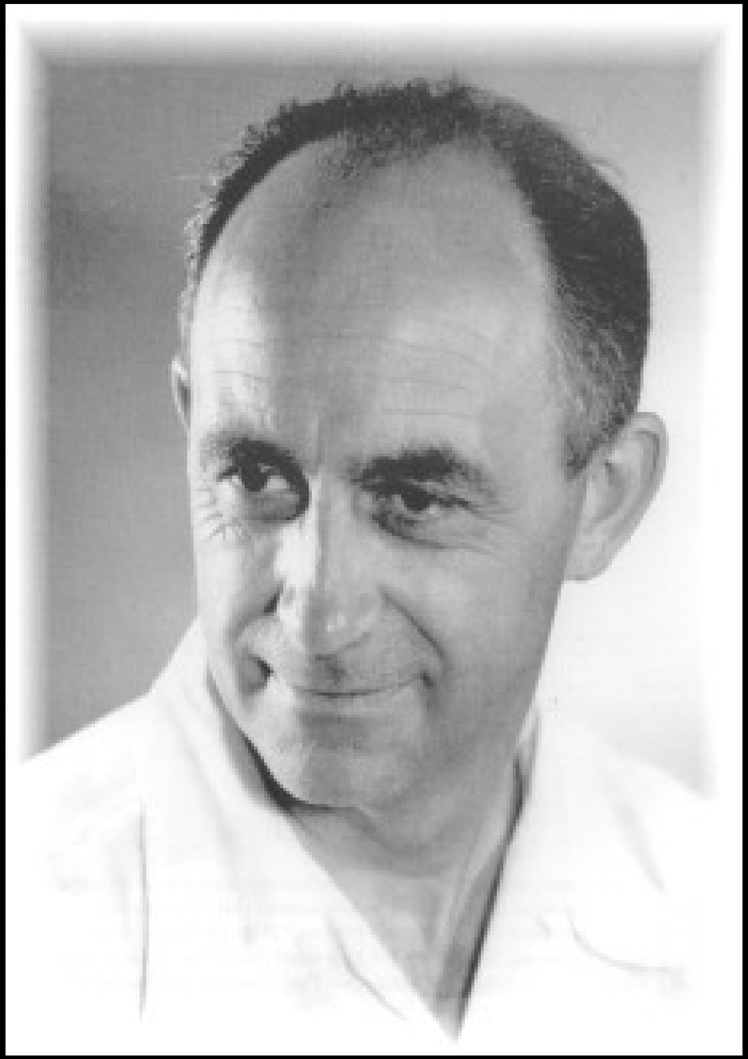
Sir Ken Robinson (1950-)


EXPERIMENT
what if
The Chessboard
The Ruler of India long ago, it is told, was so thrilled with the new game of chess that he offered its inventor any prize he wished. The inventor said, "If you would just give me enough wheat to put one grain in the first square, two in the second, four in the third, doubling the number of grains in each square across the whole chessboard, I would well be satisfied".
1 Here is a chessboard. Do you think that this was a fair prize to ask for? How many squares are there? How much wheat would you think that would be and what would the ruler do?
2 Try it. Use a formula to double the number in each square. In order to make room for larger numbers I suggest you complete the doubling in the table below.
3 By writing the doublings as powers of 2 we can see the pattern more easily. Complete the chessboard as powers of 2 in the table below. (^ means to the power. 2^5 written as 2^5 is 2 to the 5th power or 2 multiplied by itself 5 times.)
4 You can calculate the powers of two by changing each of the strings into a formula. Or you can can use the new table below to do this.
1 2 4 8 16 32 64 128
256 512 1024 2048 4096 8192 16384 32768
65536 131072 262144 524288 1048576 2097152 4194304 8388608
1.7E+07 3.4E+07 6.7E+07 1.3E+08 2.7E+08 5.4E+08 1.1E+09 2.1E+09
4.3E+09 8.6E+09 1.7E+10 3.4E+10 6.9E+10 1.4E+11 2.7E+11 5.5E+11
1.1E+12 2.2E+12 4.4E+12 8.8E+12 1.8E+13 3.5E+13 7E+13 1.4E+14
2.8E+14 5.6E+14 1.1E+15 2.3E+15 4.5E+15 9E+15 1.8E+16 3.6E+16

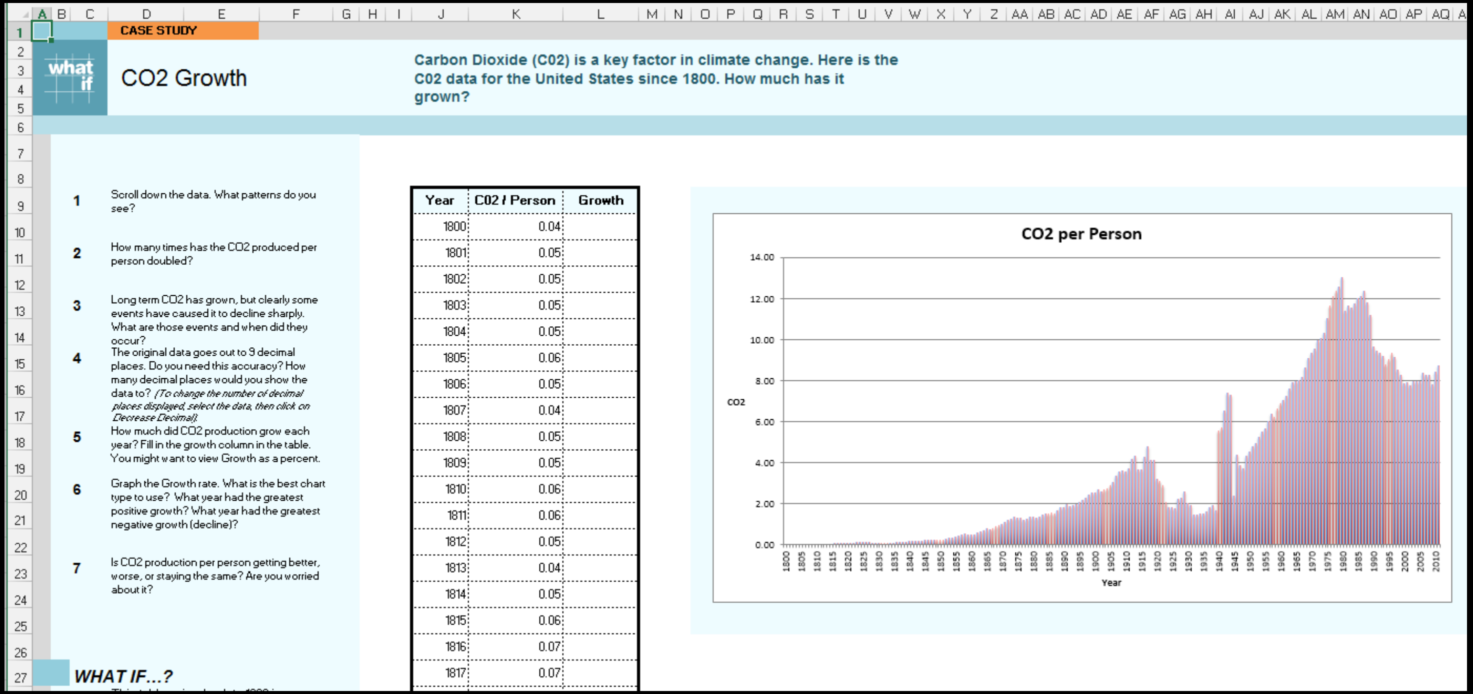

CHALLENGE
what if
String Challenge
Look up images of curve stitching or string diagrams on the Web. Pick one and see if you can use the spreadsheet to make it. The axes need not be at right angles (Cartesian). What would your curve stitching look like then.
1 Here is a different string diagram that does not have cartesian axes. Since I did not separate the ordered pairs into blocks, it has the return lines in it.
2 How did I get y-axis to tilt? Can you change the formula to change the tilt of y?
3 Change the formula to get the x-axis tilted as well? Do you like the way that changes the string diagram?
4 Use different rules and different scales to create string diagrams that are beautiful works of art on your spreadsheet.
WHAT IF...?
What if you used an entirely different function, say a quadratic function for one of the axes? How would this affect the string diagram?
x y
10 20
1 0
9.5 19
2 0
9 18
3 0
8.5 17
4 0
8 16
5 0
7.5 15
6 0
7 14
7 0
6.5 13
8 0
6 12
9 0
5.5 11
10 0
5 10
11 0
String Diagrams

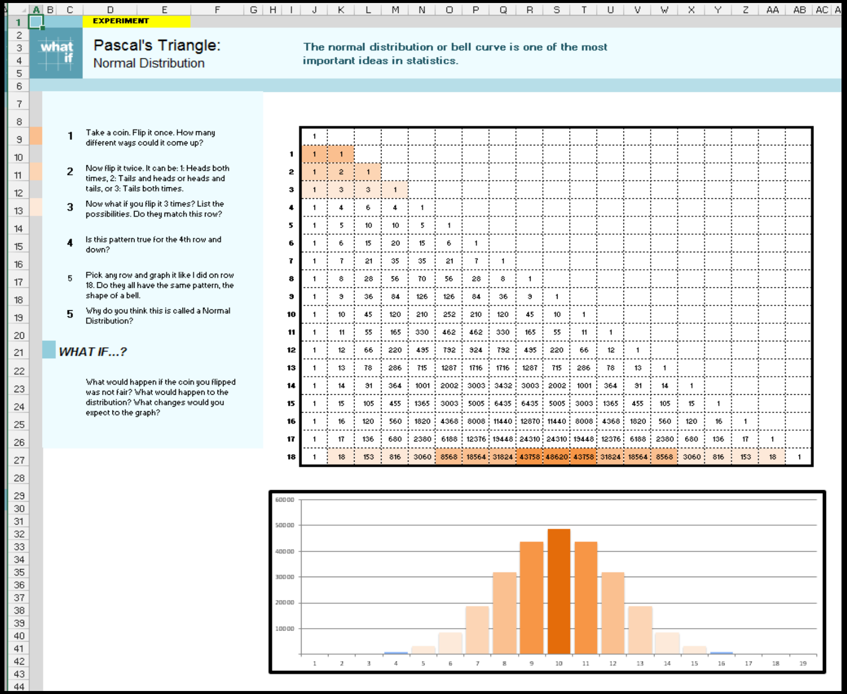

what if
Inverse of a Function.1
If you interchange the inputs and outputs of a function, you create what is called the "inverse of the function". What will this new relationship look like? Can you describe the relationship between the function and its inverse?
1 This power function f(x) has three parameters: m, b, and p. We can use it to start exploring the inverse of a function. Start by entering parameters for m, b, and p.
2 Enter parameters for x0 and Δx to create a dynamic input column and build the input (x) table. Then build the output f(x) table by entering the rule for the function and copy it down the table.
3 We linked the graph for you. Explore f(x) by changing the parameters. What patterns do you see? How do the different parameters change the patterns?
4 Create the table for the Inverse f⁻¹(x). This table should just interchange the input and output columns of f(x). What simple rules do you need in each column to display this new relationship?
5 Graph both tables. Explore both of these relationships with different parameters. Can you find a pattern?
6 Complete the table for y=x and add this function to the graph. Does this help you see some symmetry between f⁻¹(x) and f(x)?
7 Is the inverse of a function a function? Always? Sometimes? Never? (hint: Experiment with different values of p to investigate)
f(x)=mx^p+b
f(x)=2x^2+3
Parameters
m 2
b 3
p 2
x0 -6
Δx 0.5
Ordered Pairs
x f(x)
-6 75
-5.5 63.5
-5 53
-4.5 43.5
-4 35
-3.5 27.5
-3 21
-2.5 15.5
-2 11
-1.5 7.5
Inverse
f⁻¹(x) x
75 -6
63.5 -5.5
53 -5
43.5 -4.5
35 -4
27.5 -3.5
21 -3
15.5 -2.5
11 -2
7.5 -1.5
y=x
x y
-6 -6
-5.5 -5.5
-5 -5
-4.5 -4.5
-4 -4
-3.5 -3.5
-3 -3
-2.5 -2.5
-2 -2
-1.5 -1.5
f(x) y=x f'(x)
Inverse of a Function
Inverse of a Linear Function
Inverse of a Function(TE).1

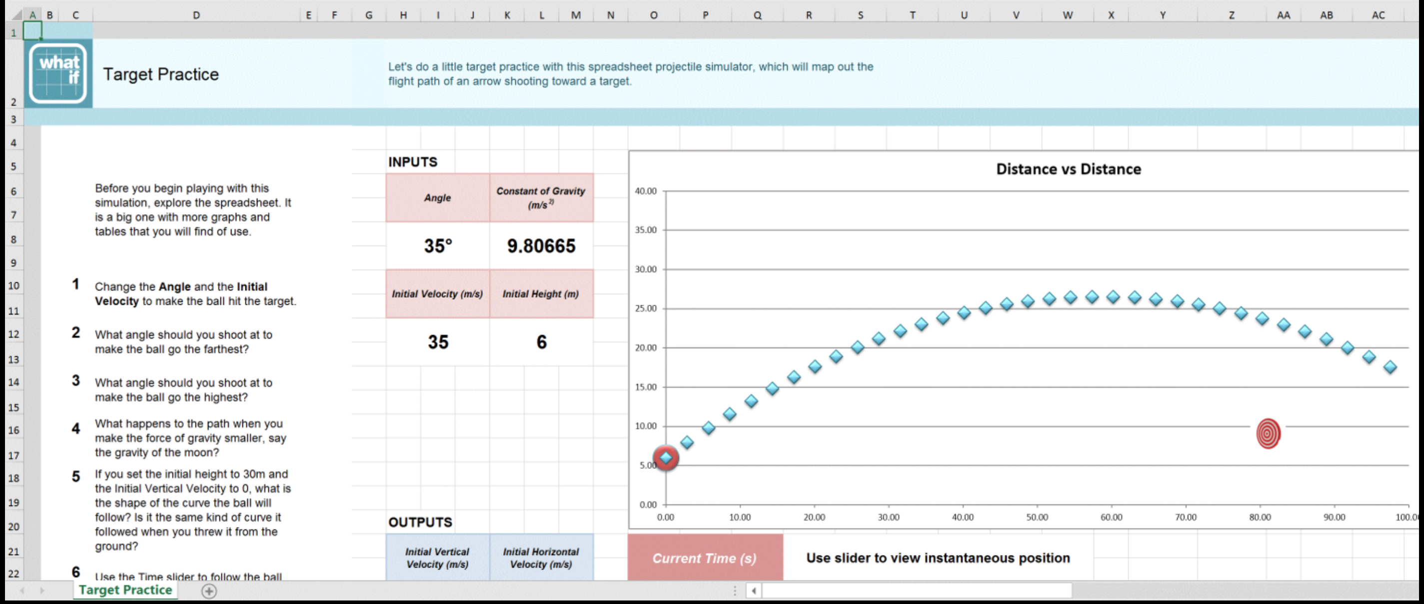
By solving interesting problems



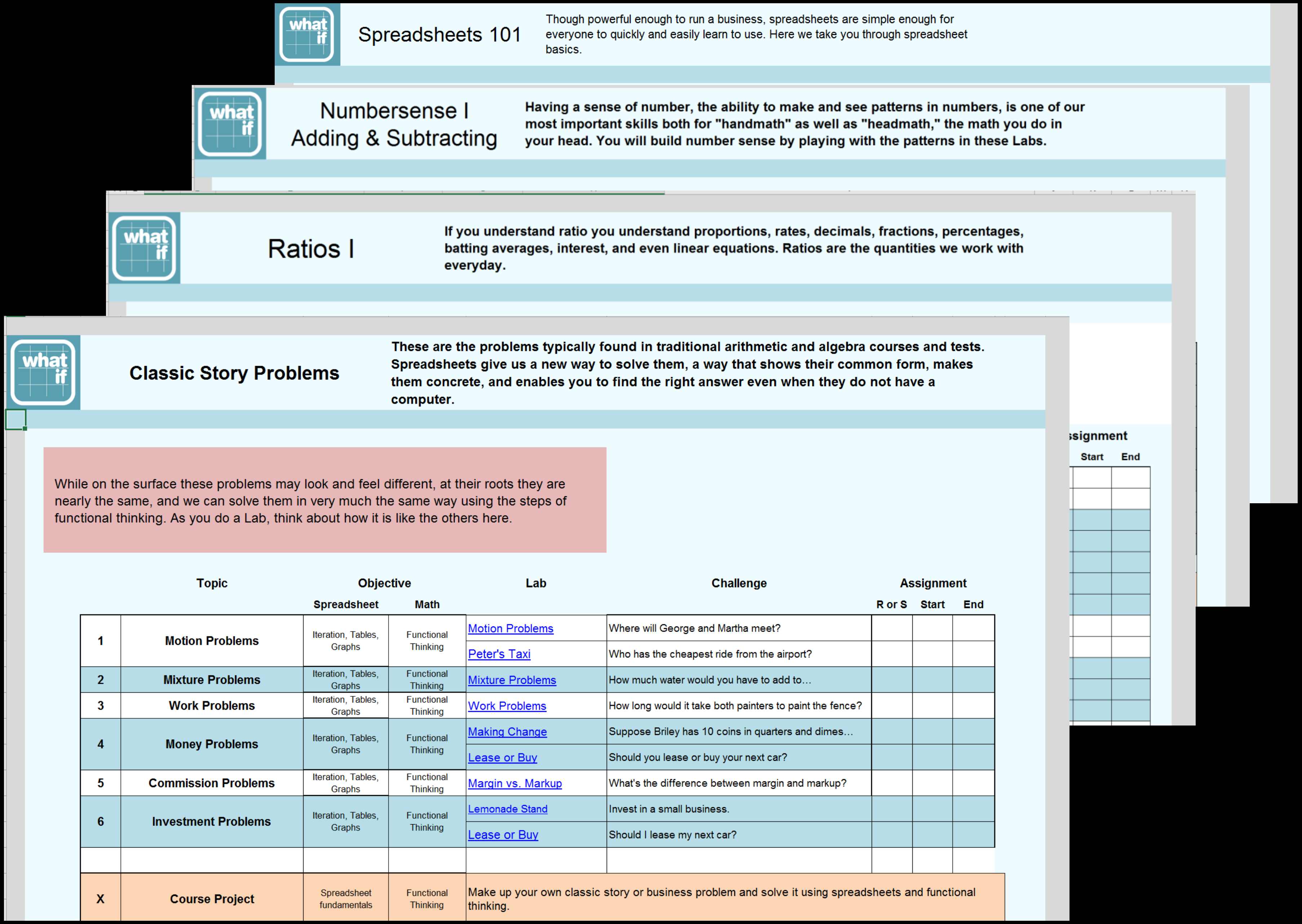
Enrico Fermi (1901-1954)


CASE STUDY
what if
CO2 Growth
Carbon Dioxide (C02) is a key factor in climate change. Here is the C02 data for the United States since 1800. How much has it grown?
1 Scroll down the data. What patterns do you see?
2 How many times has the CO2 produced per person doubled?
3 Long term CO2 has grown, but clearly some events have caused it to decline sharply. What are those events and when did they occur?
4 The original data goes out to 3 decimal places. Do you need this accuracy? How many decimal places would you show the data to? (To change the number of decimal places displayed, select the data, then click on Decrease Decimal)
5 How much did CO2 production grow each year? Fill in the growth column in the table. You might want to view Growth as a percent.
6 Graph the Growth rate. What is the best chart type to use? What year had the greatest positive growth? What year had the greatest negative growth (decline)?
7 Is CO2 production per person getting better, worse, or staying the same? Are you worried about it?
WHAT IF...?
Year | C02 / Person | Growth
1800 0.04
1801 0.05
1802 0.05
1803 0.05
1804 0.05
1805 0.06
1806 0.05
1807 0.04
1808 0.05
1809 0.05
1810 0.06
1811 0.06
1812 0.05
1813 0.04
1814 0.05
1815 0.06
1816 0.07
1817 0.07
CO2 per Person
CO2
Year


EXPERIMENT
what if
Pascal's Triangle:
Normal Distribution
The normal distribution or bell curve is one of the most important ideas in statistics.
1 Take a coin. Flip it once. How many different ways could it come up?
2 Now flip it twice. It can be: 1: Heads both times, 2: Tails and heads or heads and tails, or 3: Tails both times.
3 Now what if you flip it 3 times? List the possibilities. Do they match this row?
4 Is this pattern true for the 4th row and down?
5 Pick any row and graph it like I did on row 18. Do they all have the same pattern, the shape of a bell.
5 Why do you think this is called a Normal Distribution?
WHAT IF...?
What would happen if the coin you flipped was not fair? What would happen to the distribution? What changes would you expect to the graph?


what if
Target Practice
Let's do a little target practice with this spreadsheet projectile simulator, which will map out the flight path of an arrow shooting toward a target.
Before you begin playing with this simulation, explore the spreadsheet. It is a big one with more graphs and tables that you will find of use.
1 Change the Angle and the Initial Velocity to make the ball hit the target.
2 What angle should you shoot at to make the ball go the farthest?
3 What angle should you shoot at to make the ball go the highest?
4 What happens to the path when you make the force of gravity smaller, say the gravity of the moon?
5 If you set the initial height to 30m and the Initial Vertical Velocity to 0, what is the shape of the curve the ball will follow? Is it the same kind of curve it followed when you threw it from the ground?
6 Use the Time slider to follow the ball
INPUTS
Angle
Constant of Gravity (m/s²)
35°
9.80665
Initial Velocity (m/s)
Initial Height (m)
35
6
OUTPUTS
Initial Vertical Velocity (m/s)
Initial Horizontal Velocity (m/s)
Distance vs Distance
Current Time (s)
Use slider to view instantaneous position
Target Practice

With courses like these



Spreadsheets 101 — Though powerful enough to run a business, spreadsheets are simple enough for everyone to quickly and easily learn to use. Here we take you through spreadsheet basics.

Numbersense I Adding & Subtracting — Having a sense of number, the ability to make and see patterns in numbers, is one of our most important skills both for "handmath" as well as "headmath," the math you do in your head. You will build number sense by playing with the patterns in these Labs.

Ratios I — If you understand ratio you understand proportions, rates, decimals, fractions, percentages, batting averages, interest, and even linear equations. Ratios are the quantities we work with everyday.

Classic Story Problems — These are the problems typically found in traditional arithmetic and algebra courses and tests. Spreadsheets give us a new way to solve them, a way that shows their common form, makes them concrete, and enables you to find the right answer even when they do not have a computer.

While on the surface these problems may look and feel different, at their roots they are nearly the same, and we can solve them in very much the same way using the steps of functional thinking. As you do a Lab, think about how it is like the others here.

	Topic	Objective		Lab	Challenge	Assignment		
		Spreadsheet	Math			R or S	Start	End
1	Motion Problems	Iteration, Tables, Graphs	Functional Thinking	Motion Problems	Where will George and Martha meet?			
				Peter's Taxi	Who has the cheapest ride from the airport?			
2	Mixture Problems	Iteration, Tables, Graphs	Functional Thinking	Mixture Problems	How much water would you have to add to…			
3	Work Problems	Iteration, Tables, Graphs	Functional Thinking	Work Problems	How long would it take both painters to paint the fence?			
4	Money Problems	Iteration, Tables, Graphs	Functional Thinking	Making Change	Suppose Briley has 10 coins in quarters and dimes…			
				Lease or Buy	Should you lease or buy your next car?			
5	Commission Problems	Iteration, Tables, Graphs	Functional Thinking	Margin vs. Markup	What's the difference between margin and markup?			
6	Investment Problems	Iteration, Tables, Graphs	Functional Thinking	Lemonade Stand	Invest in a small business.			
				Lease or Buy	Should I lease my next car?			
X	Course Project	Spreadsheet fundamentals	Functional Thinking	Make up your own classic story or business problem and solve it using spreadsheets and functional thinking.				

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